International Journal of Advanced Science and Research
ISSN: 2455-4227, Impact Factor: RJIF 5.12
www.allsciencejournal.com
Volume 2; Issue 3; May 2017; Page No. 70-75

Classification of operator matrices and their application

[1] Vaishali Achesariya, *[2] Dr. Pradeep Jha, [3] Dr. Amit Parikh

[1, 2] Research Scholar, C.U.Shah University, Wadhwan, Gujarat, India
[3] Research Guide, C.U.Shah University, Wadhwan, Gujarat, India

Abstract
First we introduce the basic concepts of Pythagorean triplets and classify them into four different categories. These operator matrices demonstrate many classical properties on dealing them in connection to algebraic structural properties. In this paper our target is to introduce operator matrices (Pre, Post, and Shift) which when operator on Pythagorean and Fermat In this paper our target is to introduce operator matrices (Pre, Post, and Shift) which when operator on Pythagorean. In the case Pythagorean matrices in which the column entries are the entries of triplets (Right triangle) of consecutive integer, the shift operator matrix preserves the order and nature of original triangle. The same logic for square matrices having column (triplet) entries of fermat triangle. In some cases important algebraic properties including the result of product of n^{th} order are also mentioned.

Keywords: pythagorean triplets, pre operator, post operator, operator matrix

1. Introduction

In this paper, we have two important features to deal with Pythagorean triplets and Pythagorean matrices having column entries as odd or even, primitive or non-primitive Pythagorean triplets. We have classified operator on class 1 to 4.In this paper we have introduce Pre Operator and Post Operator matrices [1].

The final unit we have represent fermat family the operator matrices which act on such Fermat matrices as operators converting their status and order. First we introduce the basic concepts of Pythagorean triplets and classify them into four different categories. We have certain classical matrices which when operated on the matrices of Pythagorean triplets of different classes result into corresponding set of Pythagorean matrices of another classes [2].

2. Set of Pythagorean Triplets and Related Basics:

For some a ∈ N, if we find b and h ∈ N so that the relation $\mathbf{a^2 + b^2 = h^2}$ holds true [2] **(1)** The integers a, b, and h are said to from a Pythagorean Triplets we denote it by the symbol p = (a, b, h).

We have a < b < h [a, and b are two legs of a right triangle and h is the hypotenuse.]

The set of all such Pythagorean triplets is denoted by P.

i.e. P={ p = (a, b, h) |a, b, and h ∈ N}[3]. (2)

2.1 Some Important Deductions

From the above mentioned introductory concept we have some immediate observations and some deductions which are as follows:

1. The set P is an infinite set.
2. If (a, b) =1 then the triplet is known as a' Primitive' triplet. If (a, b) > 1 and an integer then it is a composite triplet.
3. For a Primitive triplet either a∈ N is odd and b ∈ N is an even integer or Vice-Versa; but in any one of the cases **h ∈ N** is an odd integer.
4. If **a** ∈ N is odd integer and an integer b such that (a, b) = 1^5 (condition forces b to be an even integer). then the triplet p = (a, b, h) ∈ P is known as an odd (primitive) triplet. The case when a∈ N is even integer (b is an odd integer), triplet p = (**a,** b, h) ∈ P is called an even (primitive) triplet.
5. Let us denote the infinite set of all odd Pythagorean triplets by P_1 and the one with even Pythagorean triplets by the symbol P_2.

   P_1 ={p(a, b, h) | a, b, h ∈ N, (a, b) = 1, a< b < h and a is odd} [3] (3)

   P_2 = { p(a, b, h) | a, b, h ∈ N, (a, b) = 1, a< b <h and a is even}[3] (4)

   In both the sets P_1 and P_2 mentioned above the condition (1**),** i.e**.** $\mathbf{a^2 + b^2 = h^2}$ holds true.
6. Let us denote the set $P_1 \cup P_2 = P^*$ and it is obvious that the set $P_3 = (P - P^*)$ is a infinite set of composite triplets. It is also clear that $P_1 \cap P_2 = \emptyset$
7. The integers of the type **a** where **a** = 2(2n + 1) ∀ n ∈ N ∪{0} cannot possess primitive triplets (For n = 0, 1, 2, 3,...) [i.e. integer a = 2, 6, 10, 14 ….cannot possess primitive triplet.
8. It is true for some sets of well- defined numbers that each number possesses more than one, can go up to eight as we have read and verified, primitive triplets. The following table indicates a few of such types.

Table 1: Type of non-possessing number of triples

Numbers of the type (a =)	Number of triplets	For i =
4(2n + 3),n∈ N (20, 28,…..)	2	2,4,
15(2n+ 5) n∈ N (105, 135,…..)	3	1,9, 125
3(2n + 9) n∈ N (33, 39,,…..)	2	1,9
555, 615…….	4	1,9,25, 225

Integers of For a given a b and h are found by $\frac{(a^2 - i^2)}{2i}$ and $\frac{(a^2 + i^2)}{2i}$ for the values of I mentioned above [4].

2.2 General Formula –Triplets

In this section we discuss some standard results for finding triplets corresponding to a given integer a ∈ N – {1, 2}.There are many standard results that help construct Pythagorean triplets but we feel and hence we have justified formulas which can explore possibilities of existence of more triplets, if they exist and can identify them[5]. For some given integer a ∈ N – {1, 2}[5].

We find,

$$b = \frac{a^2 - i^2}{2i} \text{and } h = \frac{a^2 + i^2}{2i} \text{ b and h} \in N \text{ so that for } i \in N \quad (5)$$

As discussed earlier we note that $a < b < h$, $a^2 + b^2 = h^2$ is satisfied.

The formula given in (5) holds true for all possible triplets for the integer **a** ∈ N so long as $a^2 - i^2 > 0$ and $\frac{\mathbf{a^2 - i^2}}{\mathbf{2i}}$ is an integer. The form finds all possible triplets may '**a**' be odd / even or primitive / composite [6]. [Again we note that it if 'a' is an odd integer then i = 1, 3, 5,…… and i = 2, 4, 6,…]

2.3 Theorem: The natural numbers of the form {2(2n +1) /∈ *N* } cannot possess any primitive triplet.

Proof: Let $\mathbf{a = 4n_1+2}$ for some $\mathbf{n_1 \in N}$. possess a primitive triplet **a** is an even integer. We write $\mathbf{a = 2P_1}$ where $\mathbf{P_1 = 2n_1+1}$; which is always an odd integer for $n_1 \in N$. We have $a^2 = 4P_1^2$ which an even integer.

Now by the given formula $\mathbf{b} = \frac{\mathbf{a^2 - i^2}}{\mathbf{2i}}$ which gives $\mathbf{b} = \frac{4P_1^2 - (i)^2}{2i}$ and for the first integer value for **i = 2** gives $\mathbf{b = (P_1^2 - 1)}$

As $P_1 = 2n_1+1$ and odd integer, $b = (P_1^2 - 1)$ is even integer.

i.e. $b = P_1^2 - 1 = 2m$ for some m ∈ N.

i.e. $a=2P_1$ and b= 2m. This gives GCD of $\mathbf{a=2P_1}$ and b=2m is '2' i.e. (a, b)=2.

This contradicts the assumption that $\mathbf{a = 4n_1 + 2}$ can possess some primitive triplet.

This proves that numbers of the form $\mathbf{a = 4n_1 + 2}$ cannot possess any primitive triplet.

3. Pythagorean Matrices and Notation

We now define different type of matrices whose column entries are Pythagorean triplets.

These Matrices are of the order either **3 x 3** or **3 x n** where **n > 3**. We introduce four classes of matrices[7].

3.1 Class-1

Matrices in this class possess the order **3 x 3** or **3 x n (n > 3)**.Each column entry of any member matrix of this class is an odd Pythagorean triplet. Also the first entry of successive column is in an increasing order. We denote member matrix of this class-1 by the notation $\mathbf{O_1}$. i.e. $\mathbf{O_1} \in$ **Class-1.**

We shall, in this note, follow the notation C_1 to denote the class[8].

$$\text{Let } O_1 = \begin{bmatrix} a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad \mathbf{(6)}$$

With (a_1, b, c_1), (a_2, b_2, c_2), and $(a_3, b_3, c_3) \in P_1$ [i.e. Each triplet is an odd triplet and also $\mathbf{a_1 < a_2 < a_3}$; here each a_1, a_2, and a_3 are consecutive odd integer.][8] It is the stage to recall that $\mathbf{b_k} = \frac{\mathbf{(a_k^2 - i^2)}}{\mathbf{2i}}$ and $\mathbf{h_k} = \frac{\mathbf{(a_k^2 + i^2)}}{\mathbf{2i}}$ for k=1,2,3….

Also for i=1 it assures for a primitive triplet.

$$\text{e.g. } O_{1,\,3x3} = \begin{bmatrix} 5 & 7 & 9 \\ 12 & 24 & 40 \\ 13 & 25 & 41 \end{bmatrix}$$

$$O_{2,3x4} = \begin{bmatrix} 5 & 7 & 9 & 11 \\ 12 & 24 & 40 & 60 \\ 13 & 25 & 41 & 61 \end{bmatrix}, \text{ with } O_1 \text{ and } O_2 \in C_1$$

We will, unless otherwise mentioned, consider square matrices order 3 x 3.

3.2 Class-2

Matrices in this class possess the order **3 x 3** or **3 x n (n > 3)**.Each column entry of any member matrix of this class is an even Pythagorean triplet (may be primitive or non-primitive). Also the first entry of successive column is in an increasing order. We denote member matrix of this class-2 by the notation $\mathbf{E_1}$ i.e. $\mathbf{E_1} \in$ **Class-2**

We shall, in this note, follow the notation C_2 to denote the class.

$$\text{Let } E_1 = \begin{bmatrix} a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad \mathbf{(7)}$$

With $\mathbf{(a_1, b_1, c_1), (a_2, b_2, c_2)}$, **and** $\mathbf{(a_3, b_3, c_3) \in P_2 \cup P_3}$ [i.e. Each triplet is an even triplet and also $\mathbf{a_1 < a_2 < a_3}$, here each $\mathbf{a_1, a_2}$, and $\mathbf{a_3}$ are consecutive even integer.][7] It is the stage to recall that

$\mathbf{b_k} = \frac{\mathbf{(a_k^2 - i^2)}}{\mathbf{2i}}$ and $\mathbf{h_k} = \frac{\mathbf{(a_k^2 + i^2)}}{\mathbf{2i}}$ for k=1,2,3… Also for **i = 2** it assures for a primitive triplet.

We, in general, denote Pythagorean matrix as **E**.

$$\text{e.g. } E_{1,3x3} = \begin{bmatrix} 6 & 8 & 10 \\ 8 & 15 & 24 \\ 10 & 17 & 26 \end{bmatrix} \in C_2$$

$$E_{2,3x4} = \begin{bmatrix} 8 & 10 & 12 & 14 \\ 15 & 24 & 35 & 48 \\ 17 & 26 & 37 & 50 \end{bmatrix} \in C_2$$

3.3 Class-3

Matrices in this class possess the order **3 x 3** or **3 x n (n > 3)**. Each column entry of any member matrix of this class is a Pythagorean triplet (may be primitive or non-primitive). Also the first entry of successive column is in an increasing order. We denote member matrix of this class-3 by the notation $\mathbf{S_1}$.

i.e. $\mathbf{S_1} \in$ **Class-3.**

$$\text{Let } E_1 = \begin{bmatrix} a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad \mathbf{(8)}$$

With $\mathbf{(a_1, b_1, c_1)}$, $\mathbf{(a_2, b_2, c_2)}$, and $\mathbf{(a_3, b_3, c_3) \in P}$ [i.e. Each triplet is an even triplet and also $\mathbf{a_1 < a_2 < a_3}$, here each a_1, a_2, and a_3 are consecutive integers.] It is the stage to recall that

$\mathbf{b_k}=\frac{(a_k^2 - i^2)}{2i}$ and $\mathbf{h_k}=\frac{(a_k^2 + i^2)}{2i}$ for k=1,2,3….Also for i=1 for odd value of **a** and 2 for even value of **a** so it assures for each one being aprimitive triplet. We, in general, denote Pythagorean matrix as **S.**
We shall, in this note, follow the notation **C_3** to denote the class.

e.g. $S_{1,3x3}=\begin{bmatrix}5 & 6 & 7\\ 12 & 8 & 24\\ 13 & 10 & 25\end{bmatrix} \in C_3$

$S_{2,3x4}=\begin{bmatrix}5 & 6 & 7 & 8\\ 12 & 8 & 24 & 15\\ 13 & 10 & 25 & 17\end{bmatrix} \in C_3$

3.4 Class-4
Matrices in this class possess the order **3 x 3** or **3 x n (n > 3)**.Each column entry of any member matrix of this class is a Pythagorean triplet of Fermat family. Also the first entry of successive column is in an increasing order. We denote member matrix of this class-4 by the notation **F_1**.
i.e.**$F_1 \in$.Class-4.**
We shall, in this note, follow the notation **C_4** to denote the class.

Let $F_1=\begin{bmatrix}a_1 & a_2 & a_2\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$ **(9)**

With **(a_1, b_1, c_1), (a_2, b_2, c_2),** and **(a_3, b_3, c_3)** $\in$ P [i.e. Each triplet is an odd/even triplet and also $\mathbf{a_1 < a_2 < a_3}$ **]**

$F_{1,,\ 3x3}=\begin{bmatrix}3 & 20 & 119\\ 4 & 21 & 120\\ 5 & 29 & 169\end{bmatrix} \in C_4$ $F_{2,3x4}=\begin{bmatrix}3 & 20 & 119 & 696\\ 4 & 21 & 120 & 697\\ 5 & 29 & 169 & 985\end{bmatrix} \in C_4$

4. Operator on classes 1 to 4
There are of two types Type-1 and Type-2

4.1 Type-1
Operators are by their inherent properties, known as advancing operators working on the Pythagorean matrices of a given class-i (i=1, 2, 3, 4) is denoted by the symbol **'OPi.'** They are the post operators on a given matrix of a class. Once they are post operated on a given matrix of a class the resultant is a matrix of order 3 x 3.

Type-2 (a) Operators on class-i are the pre operators and can operate on a matrix of class
3 x 3 or 3 x nof a given matrix of a class. When they (pre) operator on a given matrix, the resultant is a matrix of the same class and same order as the one of the original matrix of the given class. It is denoted by the symbol **'POi' (i=1, 2, 3, 4)**.

Type-2 (b) Operators-'Shift Operators': Operators are the shift-operators. These are the operators which operate on a matrix of a class the result is a matrix of the same order put of the some other class as decided by the operator matrix. We denote these operator by the symbol **'SOi' (i=1, 2, 3, 4)**

4.2 Type-1 Operators on classes: In this section we consider matrices of each class and, study the effect of pre and post multiplication by the operators on the matrices of the given class.

4.3 Advancing Operators: In this section we consider different post operators on the members of each class.

4.1(a) Post operator of class-1 matrix: We consider a matrix of the class-1-All column entries odd Pythagorean triplets considered in_resonantly increasing order in successive column. Then we perform post multiplication by the matrix **'OP1'[1]**.

Let $O1=\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$ with $\mathbf{b_k}=\frac{(a_i^2-i^2)}{2i}$ and $\mathbf{h_k}=\frac{(a_i^2-i^2)}{2i}$ for k= 1,2, and 3 and **i =1**

Where **a_1=2n+1**, **a_2 = 2n+3**, **a_3 = 2n+5** for a fixed **n** $\in$ **N**. Let the post operator matrix be

$OP_1=\begin{bmatrix}1 & 3 & 6\\ -3 & -8 & -15\\ 3 & 6 & 10\end{bmatrix}$

The operation $(O_1)(OP_1\ )=\begin{bmatrix}a_4 & a_5 & a_6\\ b_4 & b_5 & b_6\\ c_4 & c_5 & c_6\end{bmatrix}$ with $b_k=\frac{(a_i^2-i^2)}{2i}$ and $c_k=\frac{(a_i^2+i^2)}{2i}$ for $k = 1,2,$ and 3 and $i = 1$

Which can be easily established.
e.g. $a_2 = a_1(1) + a_2(-3) + a_3(3)$
$= (2n+1)(1) + (2n+3)(-3) + (2n+5)(3)$
$= 2n+7$
$= a_4$ for a fixed n $\in$ N

(1) This operator OP_1 will be known as the first post-super operator in the sequence of class-1 Matrix. we define odd Pythagorean matrix. We have all odd triplets members of the matrix. A particular format is

$$O1=\begin{bmatrix}a & a+2 & a+4\\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i}\\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i}\end{bmatrix}$$

$(O_1)(OP_1) = (OP_Y)_{3\times3}$ **(10)**

Where **a = 2k+1**, **i = 1** and **k = 1$\in$ N**. Now we multiply **(O_1) (OP_1)** then we get new product is

$$O1=\begin{bmatrix}a & a+2 & a+4\\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i}\\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i}\end{bmatrix}\begin{bmatrix}1 & 3 & 6\\ -3 & -8 & -15\\ 3 & 6 & 10\end{bmatrix}$$

$$=\begin{bmatrix}a+6 & a+8 & a+10\\ \frac{a^2+12a+35}{2} & \frac{a^2+16a+63}{2} & \frac{a^2+20a+99}{2}\\ \frac{a^2+12a+37}{2} & \frac{a^2+16a+65}{2} & \frac{a^2+20a+101}{2}\end{bmatrix} \quad \mathbf{(A)}$$

e.g. $O_1=\begin{bmatrix}3 & 5 & 7\\ 4 & 12 & 24\\ 5 & 13 & 25\end{bmatrix}$ $OP_1=\begin{bmatrix}1 & 3 & 6\\ -3 & -8 & -15\\ 3 & 6 & 10\end{bmatrix}$ and $(O_1)(OP_1)=\begin{bmatrix}9 & 11 & 13\\ 40 & 60 & 84\\ 41 & 61 & 85\end{bmatrix}$

(2) As it is we define Even Pythagorean matrix as (E_1). It's general form is

$$E_1=\begin{bmatrix}a & a+2 & a+4\\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i}\\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i}\end{bmatrix}$$

Where **a = 2k**, for all **k ≥ 3** and **i = 2, k = 1**.
Now we multiply (E_1) (OP_1) then we get new product is

$$E_1=\begin{bmatrix}a & a+2 & a+4\\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i}\\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i}\end{bmatrix}\begin{bmatrix}1 & 3 & 6\\ -3 & -8 & -15\\ 3 & 6 & 10\end{bmatrix}$$

$$=\begin{bmatrix}a+6 & a+8 & a+10\\ \frac{a^2+12a+32}{4} & \frac{a^2+16a+60}{4} & \frac{a^2+20a+96}{4}\\ \frac{a^2+12a+40}{4} & \frac{a^2+16a+68}{4} & \frac{a^2+20a+104}{4}\end{bmatrix} \quad \textbf{(B)}$$

$(E_1)_{3\times 3}\,(OP_1)_{3\times 3} = (EP_Y)_{3\times 3}$

$E_1=\begin{bmatrix}6 & 8 & 10\\ 8 & 15 & 24\\ 10 & 17 & 26\end{bmatrix}$ $OP_1=\begin{bmatrix}1 & 3 & 6\\ -3 & -8 & -15\\ 3 & 6 & 10\end{bmatrix}$ and $E_1\,(OP_1)=\begin{bmatrix}12 & 14 & 16\\ 35 & 48 & 63\\ 37 & 50 & 65\end{bmatrix}$

(3) This operator PO_1 will be known as first Pre operator. We mention here

$$PO_1=\begin{bmatrix}1 & -8 & 8\\ -8 & -31 & 32\\ 8 & -32 & 33\end{bmatrix}$$

$(PO_1)\,(O_1)_{3\times 4} = (OP_Y)_{3\times 4}$ Where a = 2k + 1, i = 1 and k = 1∈ N.
Now we multiply $(PO_1)\,(O_1)_{3\times 4}$

$$(O_1)_{3\times 4}=\begin{bmatrix}a & a+2 & a+4 & a+6\\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i} & \frac{(a+6)^2-i^2}{2i}\\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i} & \frac{(a+6)^2+i^2}{2i}\end{bmatrix}$$

Where a = 2k+1, i = 1 and k = 1∈ N.
Now we multiply $(PO_1)(O_1)$ then we get new product is $(OP_Y)_{3\times 4}$

$$=\begin{bmatrix}a+8 & a+10 & a+12 & a+14\\ \frac{a^2+16a+63}{2} & \frac{a^2+20a+99}{2} & \frac{a^2+24a+143}{2} & \frac{a^2+28a+195}{2}\\ \frac{a^2+16a+65}{2} & \frac{a^2+20a+101}{2} & \frac{a^2+24a+145}{2} & \frac{a^2+28a+197}{2}\end{bmatrix} \text{©}$$

This operator matrix PO_1 is pre -operator to a Pythagorean matrix. Given any odd or even Pythagorean matrix of order $3\times n, n\geq 3$ the product $(PO_1)(O_1)$ gives the next Pythagorean matrix which is of the order to that of the Pythagorean matrix.

e.g.

$PO_1=\begin{bmatrix}1 & -8 & 8\\ -8 & -31 & 32\\ 8 & -32 & 33\end{bmatrix}$ $O_1=\begin{bmatrix}3 & 5 & 7 & 9\\ 4 & 12 & 24 & 40\\ 5 & 13 & 25 & 41\end{bmatrix}$ and

$(PO_1)(O_1)=\begin{bmatrix}11 & 13 & 15 & 17\\ 60 & 84 & 112 & 144\\ 61 & 85 & 113 & 145\end{bmatrix}$

(4) This operator matrix PO_1 is pre -operator to a Pythagorean matrix. Given any even Pythagorean matrix of order $3\times n, n\geq 3$ the product $(PO_1)\,(E_1)_{3\times 4}$ gives the next even Pythagorean matrix which is the same order we mention here

$PO_1=\begin{bmatrix}1 & -8 & 8\\ -8 & -31 & 32\\ 8 & -32 & 33\end{bmatrix}$ and E_1

$$=\begin{bmatrix}a & a+2 & a+4 & a+6\\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i} & \frac{(a+6)^2-i^2}{2i}\\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i} & \frac{(a+6)^2+i^2}{2i}\end{bmatrix}$$

Where a = 2k, for all k ≥ 3 and i = 2, k = 1.
Now we multiply $(PO_1)\,(E_1)_{3\times 4}$ then we get new product is

$$(EP_Y)=\begin{bmatrix}a+8 & a+10 & a+12 & a+14\\ \frac{a^2+16a+60}{4} & \frac{a^2+20a+96}{4} & \frac{a^2+24a+140}{4} & \frac{a^2+28a+192}{4}\\ \frac{a^2+16a+48}{4} & \frac{a^2+20a+104}{4} & \frac{a^2+24a+148}{4} & \frac{a^2+28a+200}{4}\end{bmatrix} \textbf{(D)}$$

e.g. $(E_1)_{3\times 4}=\begin{bmatrix}6 & 8 & 10 & 12\\ 8 & 15 & 24 & 35\\ 10 & 17 & 26 & 37\end{bmatrix}$, $PO_1=\begin{bmatrix}1 & -8 & 8\\ -8 & -31 & 32\\ 8 & -32 & 33\end{bmatrix}$ and

$(PO_1)(E_1)=\begin{bmatrix}14 & 16 & 18 & 20\\ 48 & 63 & 80 & 99\\ 50 & 65 & 82 & 101\end{bmatrix}$

(5) If O_1 is any Pythagorean matrix having all odd triplets then the post operator OP_2 converts it in to a matrix with odd entries in to even Pythagorean triplet
$(O_1)_{3\times 3}(OP_2) = (EP_Y)_{3\times 3}$

$(O_1)=\begin{bmatrix}a & a+2 & a+4\\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i}\\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i}\end{bmatrix}$, $(OP_2)=\begin{bmatrix}\frac{15}{4} & 2 & \frac{3}{4}\\ -\frac{5}{2} & 0 & \frac{3}{2}\\ \frac{3}{4} & 0 & -\frac{1}{4}\end{bmatrix}$

now we multiply $(O_1)(OP_2)$

then we Get general form $(EP_Y)_{3\times3}$

$$= \begin{bmatrix} 2(a-1) & 2a & 2(a+1) \\ a^2-2a & a^2-1 & a^2+2a \\ a^2-2a+2 & a^2+1 & a^2+2a+2 \end{bmatrix}$$

a = 2k, k = 2,3,4.. **(E)**

e.g. $(O_1)_{3\times3} = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 12 & 24 \\ 5 & 13 & 25 \end{bmatrix}$, $(OP_2) = \begin{bmatrix} \frac{15}{4} & 2 & \frac{3}{4} \\ -\frac{5}{2} & 0 & \frac{3}{2} \\ \frac{3}{4} & 0 & -\frac{1}{4} \end{bmatrix}$

we multiply $(O_1)_{3\times3}$ $(OP_2)_{3\times3}$

$(O_1)_{3\times3}(OP_2)$ then we get $(EP_Y)_{3\times3} = \begin{bmatrix} 4 & 6 & 8 \\ 3 & 8 & 15 \\ 5 & 10 & 17 \end{bmatrix}$

(6) If E_1 is any Pythagorean matrix having all even triplets then the pre operator po_2 converts it in to a Matrix With even entries in to odd Pythagorean triplet

(PO_2) $(E_1)_{3\times3} = (OP_Y)_{3\times3}$

$PO_2 = \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ -1 & 1 & 1 \\ -1 & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$,

$(E_1)_{3\times3} = \begin{bmatrix} a & a+2 & a+4 \\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i} \\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i} \end{bmatrix}$

a = 2k, for all k ≥ 3 and i = 2, k= 1. we multiply (PO_3) $(E_1)_{3\times3}$ then we get general form of odd Pythagorean matrix

$(OP_Y)_{3\times3} = \begin{bmatrix} a-1 & a+1 & a+3 \\ \frac{a^2-2a}{2} & \frac{a^2+2a}{2} & \frac{a^2+6a+8}{2} \\ \frac{a^2-2a+2}{2} & \frac{a^2+2a+2}{2} & \frac{a^2+6a+10}{2} \end{bmatrix}$

a=2k+1,for a k ≥ 3 and i=1, k= 1. **(F)**

e.g. $\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ -1 & 1 & 1 \\ -1 & \frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 6 & 8 & 10 \\ 8 & 15 & 24 \\ 10 & 17 & 26 \end{bmatrix} = \begin{bmatrix} 5 & 7 & 9 \\ 12 & 24 & 40 \\ 13 & 25 & 41 \end{bmatrix}$

we multiply (PO_3) $(E_1)_{3\times3}$ then we get

(7) If O_1 is any Pythagorean matrix having all odd triplets then the post operator PO_3 converts it in to a Matrix With even entries in to even Pythagorean triplet Given any odd Pythagorean matrix of order $3\times n, n \geq 3$ the product $(PO_3)_{3\times3}$ $(O_1)_{3\times4}$ gives the next even Pythagorean matrix which is the Same order $(PO_3)_{3\times3}$ $(O_1)_{3\times4} = (EP_Y)_{3\times4}$

$PO_3 = \begin{bmatrix} 1 & -1 & 1 \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{2} & -1 & \frac{3}{2} \end{bmatrix}$, $(O_1)_{3\times4}$

$$= \begin{bmatrix} a & a+2 & a+4 & a+6 \\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i} & \frac{(a+6)^2-i^2}{2i} \\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i} & \frac{(a+6)^2+i^2}{2i} \end{bmatrix}$$

a=2k+1,for a k ≥ 3 and i=1, k= 1.

$(PO_3)_{3\times3}$ $(O_1)_{3\times4}$ then we get $(EP_Y)_{3\times4}$

$$= \begin{bmatrix} a+1 & a+3 & a+5 & a+7 \\ \frac{a^2+2a-3}{4} & \frac{a^2+6a+5}{4} & \frac{a^2+10a+21}{4} & \frac{a^2+14a+45}{4} \\ \frac{a^2+2a+5}{4} & \frac{a^2+6a+13}{4} & \frac{a^2+10a+29}{4} & \frac{a^2+14a+53}{4} \end{bmatrix} \textbf{(G)}$$

Given any odd Pythagorean matrix of order $3\times n, n \geq 3$ the product $(PO_3)_{3\times3}$ $(O_1)_{3\times4}$ gives the next even Pythagorean matrix $(EP_Y)_{3\times4}$ which is the Same order

e.g $PO_3 = \begin{bmatrix} 1 & -1 & 1 \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{2} & -1 & \frac{3}{2} \end{bmatrix}$, $(O_1)_{3\times4}$

$= \begin{bmatrix} 3 & 5 & 7 & 9 \\ 12 & 24 & 40 & 60 \\ 13 & 25 & 41 & 61 \end{bmatrix}$ WE have multiply

$(PO_3)_{3\times3}(O_1)_{3\times4}$ then we get $(EP_Y)_{3\times4} = \begin{bmatrix} 4 & 6 & 8 & 10 \\ 3 & 8 & 15 & 24 \\ 5 & 10 & 17 & 26 \end{bmatrix}$

(8) If E_1 is any Pythagorean matrix having all even triplets then the post operator OP_3 converts it in to a Matrix With even entries in to odd Pythagorean triplet

$(E_1)_{3\times3}$ $(OP_3) = (OP_Y)_{3\times3}$

$(E_1)_{3\times3} = \begin{bmatrix} a & a+2 & a+4 \\ \frac{a^2-i^2}{2i} & \frac{(a+2)^2-i^2}{2i} & \frac{(a+4)^2-i^2}{2i} \\ \frac{a^2+i^2}{2i} & \frac{(a+2)^2+i^2}{2i} & \frac{(a+4)^2+i^2}{2i} \end{bmatrix}$ a = 2k, for all k ≥ 3

and i = 2, k= 1., $(OP_3) = \begin{bmatrix} 0 & \frac{1}{2} & 3 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{15}{2} \\ 0 & \frac{3}{2} & 5 \end{bmatrix}$

$(OP_Y)_{3\times3} = \begin{bmatrix} \frac{a+2}{2} & \frac{a+6}{2} & \frac{a+10}{2} \\ \frac{a(a+4)}{8} & \frac{a^2+12a+32}{8} & \frac{a^2+20a+96}{8} \\ \frac{a^2+4a+8}{8} & \frac{a^2+12a+40}{8} & \frac{a^2+20a+104}{8} \end{bmatrix}$ **(H)**

e.g.$(E_1)_{3\times3} = \begin{bmatrix} 8 & 10 & 12 \\ 15 & 24 & 35 \\ 17 & 26 & 37 \end{bmatrix}$, $(OP_3) = \begin{bmatrix} 0 & \frac{1}{2} & 3 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{15}{2} \\ 0 & \frac{3}{2} & 5 \end{bmatrix}$

We are multiply $(E_1)_{3\times3}$ (OP_3) then we get $(OP_y)_{3\times3}$

$$= \begin{bmatrix} 5 & 7 & 9 \\ 12 & 24 & 40 \\ 13 & 25 & 41 \end{bmatrix}$$

(OP) is any Pythagorean matrix whose triplets is from P_2 set then the post operator matrix on Pythagorean its multiplication operation puts the result to the next matrix is convert in P_1 set with the same order

Conclusion

These operator matrices exhibit many properties in their algebraic operations. We still continue to find more of such matrices which have wide application in the field of certain infinite sequences have symmetric relation in connection with their characteristic roots and higher exponents such problems have open and we put them open to research scholars

References

1. Shah SH, Prajapati DP, Achesariya VA, Dr. Jha PJ. Classification of matrices on the Basis of Special Characteristics, International Journal of Mathematics Trends and Technology. 2015; 2:91-101, ISNN:2231-5373.
2. Shah SH, Achesariya VA, J. Jha PJ. Operators on Pythagorean Matrices, IOSR Journal of mathematics. 2015; 11, 19(2):51-60, ISSN: 2278-5728.
3. Vaishali Achesariya, Dr. Jha PJ, Three Super Operator Matrices their Properties and Applications International conference on Emerging trends in scientific research (ICETSR) C.U.Shah University, Wadhwan, India. 2015; 231-234. ISBN: 978-2-642-24819-9.
4. Trivedi RA, Bhanotar SA. Dr. Jha. PJ. Pythagorean triplets- views, analysis, and classification, IOSR journal of Mathematics, March. 2015; 11(2):54-63.e-ISSN: 2278-5728
5. Submitted by Marianne, 2012. Triples and quadruples: from Pythagoras to Fermat | plus.maths.org,https ://plus.maths.org/content/triples-and-quadruples.
6. Submitted by Marianne, 2015. Ramanujan surprises again |Plus.maths.org, https://plus.maths.org/content/triples-and-quadruples.
7. Sheth IH. Abstract Algebra, Eastern economy edition, India. 2004; 1-348, ISBN 8120317378.
8. Hazra AK. Matrix: Algebra, Calculus and Generalized Inverse, Viva Books Pvt.Ltd, First Indian Edition. 2009; l(1):75-120, ISBN: 978-81-309-0952-3.