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A Study on the Significance of Ordinary Differential Equations in Dynamic Modeling

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Abstract
Ordinary differential equations are used to describe responses of a dynamical system to all possible inputs and initial conditions. Equations which do not have a solution for some valid inputs and initial conditions do not define system’s behavior completely, and, hence, are inappropriate for use in analysis and design.

Keywords: Differential Equation, Dynamic, Variable

Introduction

A differential equation is an equation between specified derivative on an unknown function, its values, and known quantities and functions. Many physical laws are most simply and naturally formulated as ordinary differential equations. For this reason, ODEs have been studied by the greatest mathematicians and mathematical physicists since the time of Newton.

Ordinary differential equations are DEs whose unknowns are functions of a single variable; they arise most commonly in the study of dynamical systems and electrical networks. They are much easier to treat that partial differential equations, whose unknown functions depend on two or more independent variables.

An ordinary differential equation is a mathematical equation that relates some function of one or more variables with its derivatives. Ordinary differential equations arise whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time (expressed as derivatives) is known or postulated. Because such relations are extremely common, differential equations play a prominent role in many disciplines including engineering, physics, economics, and biology.

Ordinary differential equations are mathematically studied from several different perspectives, mostly concerned with their solutions the set of functions that satisfy the equation. Only the simplest differential equations admit solutions given by explicit formulas; however, some properties of solutions of a given differential equation may be determined without finding their exact form.

If a self-contained formula for the solution is not available, the solution may be numerically approximated using computers. The theory of dynamical systems puts emphasis on qualitative analysis of systems described by differential equations, while many numerical methods have been developed to determine solutions with a given degree of accuracy.

Many fundamental laws of physics and chemistry can be formulated as differential equations. In biology and economics, differential equations are used to model the behavior of complex systems. The mathematical theory of differential equations first developed together with the sciences where the equations had originated and where the results found application.

However, diverse problems, sometimes originating in quite distinct scientific fields, may give rise to identical differential equations. Whenever this happens, mathematical theory behind the equations can be viewed as a unifying principle behind diverse phenomena.

Research Work

Bernoulli’s equation is one of the best-known and widely-used equations. In the present paper, we will derive this equation from the N.–S. equations again emphasizing the ease with which numerous seemingly scattered results can be obtained once these equations are available.

Let $\varphi \in \boldsymbol{H}^{1/2}(\partial\Omega)$, $\int_{\partial\Omega} \varphi \cdot \boldsymbol{n}\, ds = 0$, $\boldsymbol{g} \in \boldsymbol{H}^1(\Omega)$, $\operatorname{div} \boldsymbol{g} = 0$ in Ω, $\boldsymbol{g}|_{\partial\Omega} = \varphi$, $\boldsymbol{f} \in \boldsymbol{L}^2(\Omega)$, $\boldsymbol{u}^* \in \boldsymbol{H}^1(\Omega)$, $\boldsymbol{u}^*|_{\partial\Omega} = \varphi$ and consider the Stokes problem to find $\boldsymbol{u}$ such that

$$\boldsymbol{u}-\boldsymbol{g}\in\boldsymbol{V},$$
$$\nu((\boldsymbol{u},\boldsymbol{v}))=(\boldsymbol{f},\boldsymbol{v})\quad\forall\,\boldsymbol{v}\in\boldsymbol{V}$$

This can be reformulated with the aid of the pressure: Find u, p such that

$$\boldsymbol{u}-\boldsymbol{u}^*\in\boldsymbol{H}_0^1(\Omega),\qquad p\in L_0^2=\left\{q\in L^2(\Omega);\int_\Omega q\,\mathrm{d}x=0\right\},$$
$$\nu((\boldsymbol{u},\boldsymbol{v}))-(p,\operatorname{div}\boldsymbol{v})=(\boldsymbol{f},\boldsymbol{v})\quad\forall\,\boldsymbol{v}\in\boldsymbol{H}_0^1(\Omega),$$
$$-(q,\operatorname{div}\boldsymbol{u})=0\qquad\forall\,q\in L_0^2(\Omega).$$

Exercise:

$$\text{Let } \boldsymbol{u}\in\boldsymbol{H}^1(\Omega),\ \boldsymbol{u}|_{\partial\Omega}=\varphi,\ \int_{\partial\Omega}\varphi\cdot\boldsymbol{n}\,\mathrm{d}S=0.\ \text{Prove that } \operatorname{div}\boldsymbol{u}=0$$
$$—\ (q,\operatorname{div}\boldsymbol{u})=0\quad\forall\,q\in L^2(\Omega)\quad(+)\quad\Leftrightarrow$$
$$—\ (q,\operatorname{div}\boldsymbol{u})=0\quad\forall\,q\in L_0^2(\Omega).\quad(*)$$

Proof

The implication ⇒ is obvious.

The implication ⇐: Let (*) hold. Then we can write

$$q\in L^2(\Omega)\longrightarrow\tilde{q}=q-\frac{1}{|\Omega|}\int_\Omega q\,\mathrm{d}x\in L_0^2(\Omega)\Rightarrow$$
$$\Rightarrow 0=(\tilde{q},\operatorname{div}\boldsymbol{u})=(q,\operatorname{div}\boldsymbol{u})-\frac{1}{|\Omega|}\int_\Omega q\,\mathrm{d}x(1,\operatorname{div}\boldsymbol{u})$$
$$(1,\operatorname{div}\boldsymbol{u})=\int_\Omega\operatorname{div}\boldsymbol{u}\,\mathrm{d}x=\int_{\partial\Omega}\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S=\int_{\partial\Omega}\varphi\cdot\boldsymbol{n}\,\mathrm{d}S=0$$
$$(+)\ \Rightarrow\ \operatorname{div}\boldsymbol{u}=0\ \text{— clear.}$$

For simplicity we assume that $N=2$, Ω is a polygonal domain, $\mathcal{T}_h$ is a tri- angulation of Ω with standard properties. This means that $K\in\mathcal{T}_h$ are closed triangles,

$$\overline{\Omega}=\cup K\in\mathcal{T}_hK$$

Over $T_h$we construct finite dimensional spaces and consider approximations

$$X_h\approx H^1(\Omega),\qquad X_{h0}\approx H_0^1(\Omega),\qquad \boldsymbol{V}_h\approx\boldsymbol{V},$$
$$\boldsymbol{X}_h\approx\boldsymbol{H}^1(\Omega),\qquad \boldsymbol{X}_{h0}\approx\boldsymbol{H}_0^1(\Omega)$$
$$M_h\approx L^2(\Omega),\qquad M_{h0}\approx L_0^2(\Omega),$$
$$\boldsymbol{X}_h,\boldsymbol{V}_h,\ldots\subset\boldsymbol{L}^2(\Omega),\ M_h\subset L^2(\Omega),M_{h0}\subset L_0^2(\Omega),$$
$$((\cdot,\cdot))_h\approx((\cdot,\cdot)),\qquad |||\cdot|||_h\approx|||\cdot|||,\qquad \boldsymbol{g}_h\approx\boldsymbol{g},\qquad \boldsymbol{u}_h^*\approx\boldsymbol{u}^*,$$
$$\operatorname{div}_h\approx\operatorname{div},b_h(\cdot,\cdot,\cdot)\approx b(\cdot,\cdot,\cdot)$$

Significance of the Study

A *dynamic* model accounts for time-dependent changes in the state of the system, while a *static* (or steady-state) model calculates the system in equilibrium, and thus is time-invariant. Dynamic models typically are represented by differential equations.

Assessing the scope of a model, that is, determining what situations the model is applicable to, can be less straightforward. If the model was constructed based on a set of data, one must determine for which systems or situations the known data is a "typical" set of data.
The question of whether the model describes well the properties of the system between data points is called interpolation and the same question for events or data points outside the observed data is called extrapolation.
As an example of the typical limitations of the scope of a model, in evaluating Newtonian classical mechanics, we can note that Newton made his measurements without advanced equipment, so he could not measure properties of particles travelling at speeds close to the speed of light. Likewise, he did not measure the movements of molecules and other small particles, but macro particles only. It is then not surprising that his model does not extrapolate well into these domains, even though his model is quite sufficient for ordinary life physics.

The dynamic model represents the time–dependent aspects of a system. It is concerned with the temporal changes in the states of the objects in a system. The main concepts are:

- State, which is the situation at a particular condition during the lifetime of an object.
- Transition, a change in the state
- Event, an occurrence that triggers transitions
- Action, an uninterrupted and atomic computation that occurs due to some event, and
- Concurrency of transitions.

A state machine models the behavior of an object as it passes through a number of states in its lifetime due to some events as well as the actions occurring due to the events. A state machine is graphically represented through a state transition diagram.

Conclusion

The aim of this paper is to furnish some results in very different areas that are linked by the common scope of giving new insight in the field of dynamics. The study has been attached with a wide range of mathematical techniques and today, this is a stimulating part of both pure an applied mathematics.

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