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Profit and behavior of two unit system with stand by

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Abstract
In this paper Profit and Behavior of Two Unit System with Stand By using Regenerative Point Graphical Technique (RPGT) is discussed. Problem is formulated and solved using RPGT. Repair and Failure are statistically independent. Expressions for system parameters i.e. availability, number of server visits and busy period of the server are evaluated to study the profit and behavior of the system for steady state. System behavior is discussed with the help of graphs and tables.

Keywords: - Availability, Base-State, RPGT, System Parameters.

1. Introduction

Considering the importance of individual units in system Kumar, J. & Malik, S. C. [1] have discussed the concept of preventive maintenance for a single unit system. Liu., R. [2], Malik, S. C. [3], Nakagawa, T. and Osaki, S. [4] have discussed reliability analysis of a one unit system with un-repairable spare units and its applications. Goel, P. & Singh J. [5], Gupta, P., Singh, J. & Singh, I.P. [6], Kumar, S. & Goel, P. [7], Gupta, V. K. [8], Chaudhary, Goel & Kumar [9] Sharma & Goel [10], Ritikesh & Goel [11] and Goyal & Goel [12] have discussed behavior with perfect and imperfect switch-over of systems using various techniques.

In this paper Profit and Behavior of Two Unit System with Stand By using Regenerative Point Graphical Technique (RPGT) is discussed. Problem is formulated and solved using RPGT. Repair and Failure are statistically independent. Expressions for system parameters i.e. availability, number of server visits and busy period of the server are evaluated to study the profit and behavior of the system for steady state. System behavior is discussed with the help of graphs and tables.

In this paper is devoted to the two unit redundant system having perfect switch over devices. For example, the Furnace system in industries, in which fuel can be supplied from different sources. Sometimes, it may be difficult to detect manually that which of the working units has failed and it involves risks that the failed unit is not switched out and the stand-by unit is not switched in successfully or the repaired unit is not put back in the system. It is also possible that the failed unit is not detected or the failed unit is not switched out and the stand-by unit is not switched in time. To deal with such a situation a switch over device is necessary which overcomes such problems and can detect and disconnect the failed unit with a high degree of precision. A switch over device to switch out the failed unit and to switch I the stand-by unit is necessary. Switch-over system is perfect, so that on the failure of an operating unit, the stand-by unit is switched in with a high degree of precision by mean of a switch over device. The switch may take action by electric relays, hydraulic valves, electric control circuits or some other devices. Switch-over is instantaneous i.e. if standby unit is on line, and main unit is repaired in the meantime, then, the main unit is switched in as on line. The system is down when any of mainstream units is failed and nothing can fail further when the system is in failed state. If the mainstream unit fails, the system is in degraded state and the failed unit is immediately put under repair. Repairs are perfect i.e. the repair facility never does any damage to the units and a repaired unit works like a new-one. Priority policy is in the order Mainstreams unit, redundant unit. The distributions of the failure times and repair times are exponential and general respectively and also different for operating and standby unit. They are also assumed to be independent of each other. The system is discussed for steady state conditions. The profit analysis of the system carried out by using some of the system characteristics as mentioned above.

2. Assumptions and Notations

The following assumptions and notations/symbols are used:

1. The distributions of the failure times and repair times are exponential and general respectively and also different for operating and standby unit. They are also assumed to be independent of each other.
2. Repairs are perfect i.e. the repair facility never does any damage to the units.
3. A repaired unit works like a new-one.
4. Priority policy is in the order Mainstreams unit, redundant unit.
5. The system is down when any of mainstream units is failed.
6. Nothing can fail further when the system is in failed state. If the mainstream unit fails, the system is in degraded state and the failed unit is immediately put under repair.
7. The system is discussed for steady state conditions.
8. If standby unit is on line, and main unit is repaired in the meantime, then, the main unit is switched in as on line.
9. Upon failure, if the repairman is busy and if standby unit is also fail, it joints the end of the queue of failed unit.
10. Switch-over system is perfect.
11. Switch-over is instantaneous. The repair of a failed unit starts at once.

pr/pf : Probability/transition probability factor.

$\overline{cycle}$: a circuit formed through un-failed states.

k-cycle : a circuit (may be formed through regenerative or non-regenerative/failed states) whose terminals are at the regenerative state k.

k-$\overline{cycle}$: a circuit (may be formed through only un-failed regenerative/non- regenerative states) whose terminals are at the regenerative state k.

$\left(i \xrightarrow{s_r} j\right)$: r-th directed simple path from i-state to j-state; r takes positive integral values for different paths from i-state to j-state.

$\left(\xi \xrightarrow{sff} i\right)$: a directed simple failure free path from ξ-state to i-state.

$V_{k,k}$:*pf* of the state k reachable from the terminal state k of the k-cycle.

$V_{\overline{k,k}}$: *pf* of the state k reachable from the terminal state k of the k-$\overline{cycle}$.

f_j : fuzziness measure of the j-state.

λ_1/λ_2 : constant failure rate of the mainstreams unit 'A'/ redundant unit.

λ : constant failure rate of the mainstreams unit 'B'.

$g(t)/G(t)$: probability density function/cumulative distribution function of the repair-time of the mainstreams unit 'A'.

$h(t)/H(t)$: probability density function/cumulative distribution function of the repair-time of the redundant unit 'A''.

$f(t)/F(t)$: probability density function/cumulative distribution function of the repair-time of the unit 'B'.

A/a : Unit in the operative state/ failed state.

$(A')/\ a'$: Unit in the standby state/ failed state.

B/b : Unit in the operative state/ failed state.

S_0 = AB(A') $S_1 = Ab(A')$ S_2 = aBA' S_3 = abA'

S_4 = aBa' $S_5 = ABa'$ $S_6 = Aba'$

States $S_{0,}S_{1,}S_{2,}S_3, S_4, S_5$ *and* S_6 are regenerative states. The possible transitions between states along with transition time c.d.f.'s are shown in Fig. 1

3 Transition Diagram of the System

Following the above assumptions and notations, the transition diagram of the system are shown in Fig. 1.

Table 1

State	Symbol
Regenerative state/point	•
Up-state	○
Failed state	□
Degenerated/Reduced state	⬭

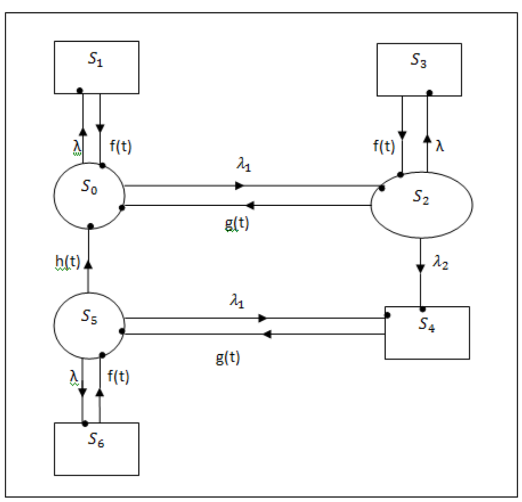

Fig. 1

4. Evaluation of Parameters of the System

The key parameters (under steady state conditions) of the system are evaluated by determining a 'base-state' and applying RPGT. The MTSF is determined w.r.t. the initial state '0' and the other parameters are obtained by using base-state.

4.1 Determination of base-state

From the transition diagram (Fig. 1), The Primary, Secondary, Tertiary circuits at all vertices are shown in Table 2

Table 2: Primary, Secondary, Tertiary circuits at a Vertex

Vertex i	(CL1)	(CL2)	(CL3)	(CL4)
0	{0,1,0} {0,2,0} {0,2,4,5,0}	{2,3,2} {2,3,2} {4,5,4} {5,6,5}	{5,6,5}	Nil
1	{1,0,1}	{0,2,0} {0,2,4,5,0}	{2,3,2} {2,3,2} {4,5,4} {5,6,5}	{5,6,5}
2	{2,0,2} {2,3,2} {2,4,5,0,2}	{0,1,0} {4,5,4} {5,6,5} {0,1,0}	{5,6,5}	Nil
3	{3,2,3}	{2,0,2} {2,4,5,0,2}	{0,1,0} {4,5,4} {5,6,5} {0,1,0}	{5,6,5}
4	{4,5,4} {4,5,0,2,4}	{5,6,5} {5,6,5} {0,1,0} {0,2,0} {2,3,2}	{2,3,2}	Nil
5	{5,6,5} {5,4,5} {5,0,2,4,5}	{0,1,0} {0,2,0} {2,3,2}	{2,3,2}	Nil
6	{6,5,6}	{5,6,5} {5,0,2,4,5}	{0,1,0} {0,2,0} {2,3,2}	{2,3,2}

In the transition diagram of Fig. 1, there are three, one, three, one, two, three and one primary circuits at the vertices 0,1, 2, 3, 4, 5 & 6 respectively. As there are three primary circuits associated each of the vertices 0, 2 & 5.So,any of these can be the base-state of the system. Now, the distinct secondary circuits along all the simple paths from the vertex '0' to all the vertices are: {2, 3, 2},{4, 5, 4},{5, 6, 5}. There are only one i.e. {5, 6, 5} tertiary circuit and no more higher level circuits along the paths from the vertex '0'. Similarly there are three, one and no primary, secondary and tertiary circuits respectively from the vertex '2' and '5'. We choose the vertex '0' as a base-state.

Table 3: Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State '0')

Vertex j	$\left(0 \xrightarrow{S_r} j\right)$: (P0)	(P1)	(P2)	(P3)
1	$\left(0 \xrightarrow{S_1} 1\right)$:{0,1}	Nil	Nil	Nil
2	$\left(0 \xrightarrow{S_1} 2\right)$:{0,2}	{2,3,2}	Nil	Nil
3	$\left(0 \xrightarrow{S_1} 3\right)$:{0,2,3}	{2,3,2}	Nil	Nil
4	$\left(0 \xrightarrow{S_1} 4\right)$:{0,2,4}	{2,3,2} {4,5,4}	{5,6,5}	Nil
5	$\left(0 \xrightarrow{S_1} 5\right)$:{0,2,4,5}	{2,3,2} {4,5,4} {5,6,5}	{5,6,5}	Nil
6	$\left(0 \xrightarrow{S_1} 6\right)$:{0,2,4,5,6}	{2,3,2} {4,5,4} {5,6,5}	{5,6,5}	Nil

4.2 Transition Probabilities and the Mean Sojourn Times

Table 4: Transition Probabilities

$q_{i,j}(t)$	$p_{i,j} = q^{*}_{i,j}(0)$
$q_{0,1}(t) = \lambda e^{-(\lambda+\lambda_1)t}$ $q_{0,2}(t) = \lambda_1 e^{-(\lambda+\lambda_1)t}$	$p_{0,1} = \frac{\lambda}{\lambda+\lambda_1}$ $p_{0,3} = \frac{\lambda_1}{\lambda+\lambda_1}$
$q_{1,0}(t) = f(t)$	$p_{1,0} = f^{*}(0)$
$q_{2,0}(t) = g(t)e^{-(\lambda+\lambda_2)t}$ $q_{2,4}(t) = \lambda_2 e^{-(\lambda+\lambda_2)t}\bar{G}(t)$ $q_{2,3}(t) = \lambda e^{-(\lambda+\lambda_2)t}\bar{G}(t)$	$p_{2,0} = g^{*}(\lambda+\lambda_2)$ $p_{2,4} = \frac{\lambda_2}{\lambda+\lambda_2}\{1 - g^{*}(\lambda+\lambda_2)\}$ $p_{2,3} = \frac{\lambda}{\lambda+\lambda_2}\{1 - g^{*}(\lambda+\lambda_2)\}$
$q_{3,2}(t) = f(t)$	$p_{3,2} = f^{*}(0)$
$q_{4,5}(t) = g(t)$	$p_{4,5} = g^{*}(0)$
$q_{5,0}(t) = h(t)e^{-(\lambda+\lambda_1)t}$ $q_{5,4}(t) = \lambda_1 e^{-(\lambda+\lambda_1)t}\bar{H}(t)$ $q_{5,6}(t) = \lambda e^{-(\lambda+\lambda_1)t}\bar{H}(t)$	$p_{5,0} = h^{*}(\lambda+\lambda_1)$ $p_{5,4} = \frac{\lambda_1}{\lambda+\lambda_1}\{1 - h^{*}(\lambda+\lambda_1)\}$ $p_{5,6} = \frac{\lambda}{\lambda+\lambda_1}\{1 - h^{*}(\lambda+\lambda_1)\}$
$q_{6,5}(t) = f(t)$	$p_{6,5} = f^{*}(0)$

It can be easily verified that;
$\boldsymbol{p_{0,1} + p_{0,2} = 1;\ p_{1,0} = 1;\ p_{2,0} + p_{2,3} + p_{2,4} = 1;\ p_{3,2} = f^{*}(0) = 1;\ p_{4,5} = g^{*}(0) = 1}$
$\boldsymbol{p_{5,0} + p_{5,4} + p_{5,6} = 1\ ;\ p_{6,5} = f^{*}(0) = 1}$

Table 5: Mean Sojourn Times

$R_i(t)$	$\mu_i = R_i^{*}(0).$
$R_0(t) = e^{-(\lambda+\lambda_1)t}$	$\mu_0 = \frac{1}{\lambda+\lambda_1}$
$R_1(t) = \bar{F}(t)$	$\mu_1 = -f^{*\prime}(0)$
$R_2(t) = e^{-(\lambda+\lambda_2)t}\bar{G}(t)$	$\mu_2 = \frac{1 - g^{*}(\lambda+\lambda_2)}{(\lambda+\lambda_2)}$
$R_3(t) = \bar{F}(t)$	$\mu_3 = -f^{*\prime}(0)$
$R_4(t) = \bar{G}(t)$	$\mu_4 = -g^{*\prime}(0)$
$R_5(t) = e^{-(\lambda+\lambda_1)t}\bar{H}(t)$	$\mu_5 = \frac{1 - h^{*}(\lambda+\lambda_1)}{(\lambda+\lambda_1)}$
$R_6(t) = \bar{F}(t)$	$\mu_6 = -f^{*\prime}(0)$

4.3 Evaluation of Parameters

The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated, by applying Regenerative Point Graphical Technique (RPGT) and using '0' as the base-state of the system as under:
The transition probability factors of all the reachable states from the base state '0' are:

$$V_{0,0} = \left[(0,1,0) + \frac{(0,2,0)}{1-L_2} + \frac{(0,2,4,5,0)}{(1-L_2)\left(1-\frac{L_4}{1-L_5}\right)(1-L_5)}\right] = 1 \qquad V_{0,1} = (0,1) = p_{0,1}$$

$$V_{0,2} = \frac{(0,2)}{1-L_2} = \frac{p_{0,2}}{1-p_{2,3}} \qquad V_{0,3} = \frac{(0,2,3)}{1-L_2} = \frac{p_{0,2}p_{2,3}}{1-p_{2,3}}$$

$$V_{0,4} = \frac{(0,2,4)}{(1-L_2)\left(1-\frac{L_4}{1-L_5}\right)} = \frac{p_{0,2}p_{2,4}(1-p_{5,6})}{p_{5,0}(1-p_{2,3})} \qquad V_{0,5} = \frac{(0,2,4,5)}{(1-L_2)\left(1-\frac{L_4}{1-L_5}\right)(1-L_5)} = \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})}$$

$$V_{0,6} = \frac{(0,2,4,5,6)}{(1-L_2)\left(1-\frac{L_4}{1-L_5}\right)(1-L_5)} = \frac{p_{0,2}p_{2,4}p_{5,6}}{p_{5,0}(1-p_{2,3})}$$

Where, $1 - L_2 = 1 - \{2,3,2\} = 1 - p_{2,3}p_{3,2} = 1 - p_{2,3}$

$1 - L_4 = 1 - \{4,5,4\} = 1 - p_{4,5}p_{5,4} = 1 - p_{5,4}$
$1 - L_5 = 1 - \{5,6,5\} = 1 - p_{5,6}p_{6,5} = 1 - p_{5,6}$
$1 - L_4 - L_5 = 1 - \{4,5,4\} - \{5,6,5\} = 1 - p_{4,5}p_{5,4} - p_{5,6}p_{6,5} = 1 - p_{5,4} - p_{5,6} = p_{5,0}$

(a). Availability of the system: From Fig. 1, the regenerative states, at which the system is available are: j = 0,2 & 5 and the regenerative states are i = 0 to 6, for 'ξ' = '0'

$$A_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\} f_j.\mu_j}{\prod_{k_1\neq\xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2\neq\xi}\{1-V_{k_2,k_2}\}}\right\}\right] = \left[\sum_j V_{\xi,j}\ .f_j.\mu_j\right] \div \left[\sum_i V_{\xi,i}\ .\mu_i^1\right]$$

$$= \left[V_{0,0}\ .f_0.\mu_0 + V_{0,2}\ .f_2.\mu_2 + V_{0,5}\ .f_5.\mu_5\right]$$
$$\div \left[V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1 + V_{0,5}\mu_5^1 + V_{0,6}\mu_6^1\right]$$

$$=[f_0\mu_0 + \frac{p_{0,2}}{1-p_{2,3}} f_1\ \mu_1 + \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})} f_5\mu_5] \div [\mu_0^1\ + p_{0,1}\mu_1^1 + \frac{p_{0,2}}{1-p_{2,3}}\mu_2^1 + \frac{p_{0,2}p_{2,3}}{1-p_{2,3}}\mu_3^1$$

$$+ \frac{p_{0,2}p_{2,4}(1-p_{5,6})}{p_{5,0}(1-p_{2,3})}\mu_4^1 + \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})}\mu_5^1 + \frac{p_{0,2}p_{2,4}p_{5,6}}{p_{5,0}(1-p_{2,3})}\mu_6^1]$$
$$= N_0\ \div\ D_0$$

Where,

$$N_0 = [f_0\mu_0 + \frac{p_{0,2}}{1-p_{2,3}} f_1\ \mu_1 + \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})} f_5\mu_5]$$

$$D_0 = [\mu_0^1\ + p_{0,1}\mu_1^1 + \frac{p_{0,2}}{1-p_{2,3}}\mu_2^1 + \frac{p_{0,2}p_{2,3}}{1-p_{2,3}}\mu_3^1 + \frac{p_{0,2}p_{2,4}(1-p_{5,6})}{p_{5,0}(1-p_{2,3})}\mu_4^1 + \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})}\mu_5^1$$
$$+ \frac{p_{0,2}p_{2,4}p_{5,6}}{p_{5,0}(1-p_{2,3})}\mu_6^1]$$

$$A_0 = N_1\ \div\ D_1$$
$$N_1 = [p_{5,0}(1-p_{2,3})\mu_0 + p_{0,2}(p_{5,0}\mu_2 + p_{2,4}\mu_5)];\left(f_j\ = 1\ \forall\ \text{j}\right)$$

$$D_1 = [p_{5,0}(1-p_{2,3})(\mu_0 + p_{0,1}\mu_1) + p_{0,2}p_{5,0}(\mu_2 + p_{2,3}\mu_3)$$

$$+ p_{0,2}p_{2,4}\{(1-p_{5,6})\mu_4 + \mu_5 + p_{5,6}\mu_6\}]\ ;(\ \mu_j^1 = \mu_j\ \forall\ \text{j})$$

(b). Busy period of the Server: From Fig. 1, the regenerative states where Server is busy while doing repairs are: j = 1,2,3,4,5,6; the regenerative states are: i = 0 to 6, for 'ξ' = '0'

$$B_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}.\eta_j}{\prod_{k_1\neq\xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2\neq\xi}\{1-V_{k_2,k_2}\}}\right\}\right] = \left[\sum_j V_{\xi,j}\ .\eta_j\right] \div \left[\sum_i V_{\xi,i}\ .\mu_i^1\right]$$

$$= \left[V_{0,1}\ .\eta_1 + V_{0,2}\ .\eta_2 + V_{0,3}.\eta_3 + V_{0,4}.\eta_4 + V_{0,5}.\eta_5 + V_{0,6}.\eta_6\right]$$

$$\div \left[V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1 + V_{0,5}\mu_5^1 + V_{0,6}\mu_6^1\right]$$

$$=[\ p_{0,1}\eta_1 + \frac{p_{0,2}}{1-p_{2,3}}\eta_2 + \frac{p_{0,2}p_{2,3}}{1-p_{2,3}}\eta_3 + \frac{p_{0,2}p_{2,4}(1-p_{5,6})}{p_{5,0}(1-p_{2,3})}\eta_4 + \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})}\eta_5 + \frac{p_{0,2}p_{2,4}p_{5,6}}{p_{5,0}(1-p_{2,3})}\eta_6]$$

$$\div [\mu_0^1\ + p_{0,1}\mu_1^1 + \frac{p_{0,2}}{1-p_{2,3}}\mu_2^1 + \frac{p_{0,2}p_{2,3}}{1-p_{2,3}}\mu_3^1 + \frac{p_{0,2}p_{2,4}(1-p_{5,6})}{p_{5,0}(1-p_{2,3})}\mu_4^1 + \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})}\mu_5^1$$
$$+ \frac{p_{0,2}p_{2,4}p_{5,6}}{p_{5,0}(1-p_{2,3})}\mu_6^1]$$

$$= N_{00} \div\ D_0$$

Where,

$$N_{00} = [\ p_{0,1}\eta_1 + \frac{p_{0,2}}{1-p_{2,3}}\eta_2 + \frac{p_{0,2}p_{2,3}}{1-p_{2,3}}\eta_3 + \frac{p_{0,2}p_{2,4}(1-p_{5,6})}{p_{5,0}(1-p_{2,3})}\eta_4 + \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})}\eta_5 + \frac{p_{0,2}p_{2,4}p_{5,6}}{p_{5,0}(1-p_{2,3})}\eta_6]$$
$$B_0 = N_{01}\ \div\ D_1$$

$$N_{01} = [p_{5,0}(1-p_{2,3})p_{0,1}\mu_1 + p_{0,2}p_{5,0}(\mu_2 + p_{2,3}\mu_3) + p_{0,2}p_{2,4}\{(1-p_{5,6})\mu_4 + \mu_5$$
$$+ p_{5,6}\mu_6\}]\ ;(\ \eta_j = \mu_j\ \forall\ \text{j})$$

$D_1 = [p_{5,0}(1-p_{2,3})(\mu_0 + p_{0,1}\mu_1) + p_{0,2}p_{5,0}(\mu_2 + p_{2,3}\mu_3)$

$+p_{0,2}p_{2,4}\{(1-p_{5,6})\mu_4 + \mu_5 + p_{5,6}\mu_6\}]$;($\mu_j^1 = \mu_j$ ∀ j)

(c). Expected number of Server's visits: From Fig. 1, the regenerative states where the Server visits (afresh) for repairs of the system are: j = 1,2; the regenerative states are: i = 0 to 6, for 'ξ' = '0'

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi \xrightarrow{sr} j)\}}{\prod_{k_1 \neq \xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,sr}\left\{\frac{\{pr(\xi \xrightarrow{sr} i)\}.\mu_i^1}{\prod_{k_2 \neq \xi}\{1-V_{k_2,k_2}\}}\right\}\right] = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i} \,.\, \mu_i^1\right]$$

$$= (V_{0,1} + V_{0,2}) \div \left[V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1 + V_{0,5}\mu_5^1 + V_{0,6}\mu_6^1\right]$$

$$= [p_{0,1} + \frac{p_{0,2}}{1-p_{2,3}}] \div [\mu_0^1 + p_{0,1}\mu_1^1 + \frac{p_{0,2}}{1-p_{2,3}}\mu_2^1 + \frac{p_{0,2}p_{2,3}}{1-p_{2,3}}\mu_3^1 + \frac{p_{0,2}p_{2,4}(1-p_{5,6})}{p_{5,0}(1-p_{2,3})}\mu_4^1$$

$$+ \frac{p_{0,2}p_{2,4}}{p_{5,0}(1-p_{2,3})}\mu_5^1 + \frac{p_{0,2}p_{2,4}p_{5,6}}{p_{5,0}(1-p_{2,3})}\mu_6^1]$$

$V_0 = N_{02} \div D_1$ $\qquad$ $N_{02} = [p_{5,0}\{p_{0,1}(1-p_{2,3}) + p_{0,2}\}]$

$D_1 = [p_{5,0}(1-p_{2,3})(\mu_0 + p_{0,1}\mu_1) + p_{0,2}p_{5,0}(\mu_2 + p_{2,3}\mu_3)$

$+p_{0,2}p_{2,4}\{(1-p_{5,6})\mu_4 + \mu_5 + p_{5,6}\mu_6\}]$;($\mu_j^1 = \mu_j$ ∀ j)

5. Particular Case

Let us take; $g(t) = \alpha e^{-\alpha t}$, $h(t) = \beta e^{-\beta t}$, $f(t) = \omega e^{-\omega t}$

We have,

$p_{0,1} = \frac{\lambda}{\lambda+\lambda_1}, p_{0,2} = \frac{\lambda_1}{\lambda+\lambda_1}, p_{1,0} = 1, p_{2,0} = \frac{\alpha}{\alpha+\lambda+\lambda_2}, p_{2,3} = \frac{\lambda}{\alpha+\lambda+\lambda_2}, p_{2,4} == \frac{\lambda_2}{\alpha+\lambda+\lambda_2}$

$, p_{3,2} = 1, p_{4,5} = 1, p_{5,0} = \frac{\beta}{\beta+\lambda+\lambda_1}, p_{5,4} = \frac{\lambda_1}{\beta+\lambda+\lambda_1}, p_{5,6} = \frac{\lambda}{\beta+\lambda+\lambda_1}, p_{6,5} = 1$

$\mu_0 = \frac{1}{\lambda+\lambda_1}, \mu_1 = \frac{1}{\omega}, \mu_2 = \frac{1}{\alpha+\lambda+\lambda_2}, \mu_3 = \frac{1}{\omega}, \mu_4 = \frac{1}{\alpha}, \mu_5 = \frac{1}{\beta+\lambda+\lambda_1}, \mu_6 = \frac{1}{\omega}$

By using these results, we get the following:

6. Special Cases

For Warm Stand-by : $0 < \lambda_2 < \lambda_1$

For Hot Stand-by : $\lambda_2 = \lambda_1$

The corresponding results from the **Section 5.6** are obtained.

7. Profit Function of the System:

The Profit analysis of the system can be done by using the profit function:

$P_0 = C_1.A_0 - C_2.B_0 - C_3.V_0$

Where, C_1 = Revenue per unit of time the system is available.

C_2 = Cost per unit time the server remains busy for the repairs.

C_3 = Cost per visit of the server.

Particularly $C_1 = 10$, $C_2 = C_3 = 1$

$P_0 = C_1A_0 - C_2B_0 - C_3V_0$

Table 6

α / λ	α = 0.80	α = 0.85	α = 0.90	α = 0.95	α = 1.00
λ = 0.005	9.8490	9.8490	9.8490	9.8491	9.8491
λ = 0.006	9.8479	9.8480	9.8480	9.8480	9.8481
λ = 0.007	9.8468	9.8469	9.8470	9.8470	9.8470
λ = 0.008	9.8458	9.8458	9.8459	9.8460	9.8460
λ = 0.009	9.8447	9.8448	9.8449	9.8449	9.8450
λ = 0.01	9.8437	9.8438	9.8438	9.8439	9.8439

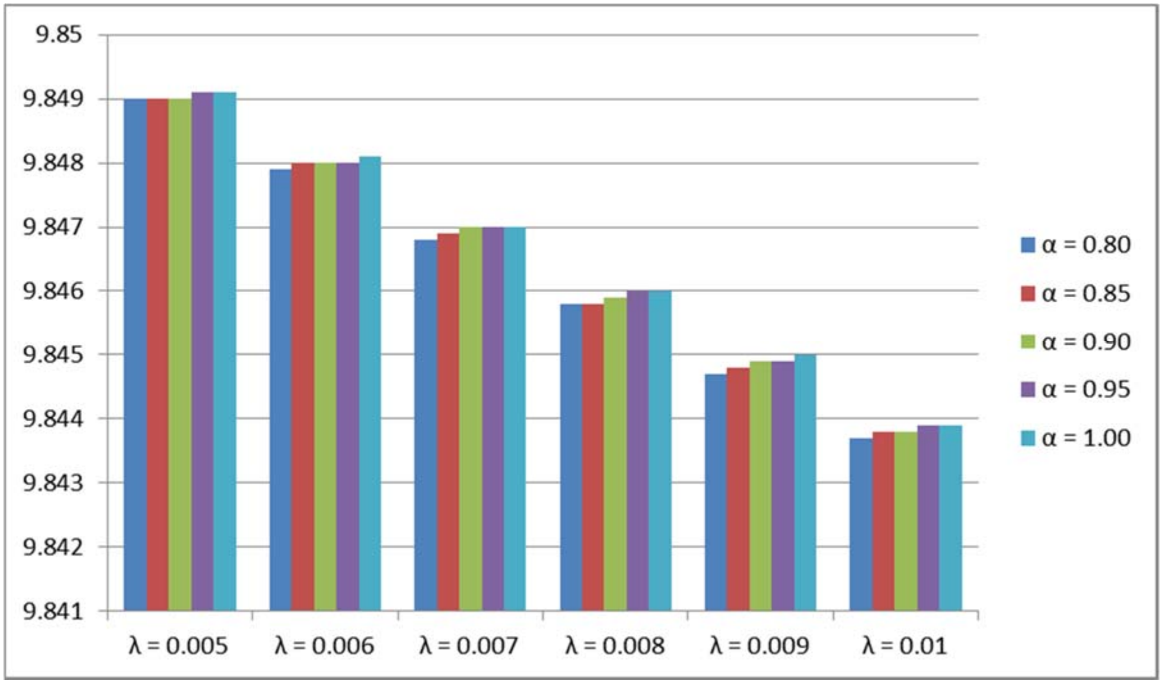

Fig 2.

8. Analytical Discussion
The following tables, graphs and conclusions are obtained for:

8.1 Availability (A_0) vs. the Repair Rate (α):
On taking
λ_2= 0.005;$\lambda = 0.01$; $\beta = 0.80$; $\omega = 0.80$.
The Availability of the system is calculated for different values of the Failure Rate (λ_1) by taking λ_1 = 0.005, 0.006, 0.007, 0.008, 0.009 and 0.01 and for different values of the repair rate (α) by taking α = 0.80, 0.85, 0.90, 0.95 and 1.0. The data so obtained are shown in Table 7.

Table 7

λ_1	A_0 (α=0.80)	A_0 (α=0.85)	A_0 (α=0.90)	A_0 (α=0.95)	A_0 (α=1.0)
0.005	0.987616	0.987621	0.987624	0.987627	0.987630
0.006	0.987611	0.987614	0.987619	0.987622	0.987626
0.007	0.987601	0.987608	0.987613	0.987617	0.987621
0.008	0.987593	0.987602	0.987607	0.987611	0.987615
0.009	0.987586	0.987593	0.987600	0.987606	0.987611
0.01	0.987578	0.987588	0.987595	0.987601	0.987606

Table 7 shows the behavior of the Availability (A_0) vs. the Repair Rate (α) of the Unit of the System for different values of the Failure Rate (λ_1). It is concluded that Availability increases with increase in the values of the Repair Rate (α).

9. Conclusion
From the table and graph for profit function we see that for a unit cost there is a profit of almost five times the cost, it increases with the increase of repair rates and decreases with the increase in failure rates, which should be so practically.

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