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The matrix, the determinant and the inverse of matrices using adjoint method

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Abstract

In this paper work we study the how to solve matrix by determine its determinant as a scaler, the adjoint, and inverse of non – singular 2 x 2 and 3 x 3 matrices by calculating their co- factors. Some computational problems and results were carry out to understand the concept.

Keywords: matrix, determinant, non - singular, adjoint and inverse

1. Introduction

The inverse of a 2 x 2 and 3 x 3 non – singular matrix i.e a matrix whose determinant is non zero can be calculated by either the adjoint method or using elementary operation i.e by Gauss Jordan Method. The determinant is then obtained by subtracting the product of the elements in the ordinary diagonal from the product of the elements in the leading diagonal. After obtaining the determinant, we will find Adjoint of matrix and then finally inverse of the matrix can be find by mulitiplication of Adjoint of the matrix with reciprocal of the determinant of the given matrix. (Ref. Celia, C.W; Nice A.T.F; Elliot. K.F. (1987) [1], Ilori S.A. and Akinyele O. (1986) [2], Lipchitz Seymour. (1988) [3], Meyer, Carl D. (2001) [4], Poole, David (2006) [5], Satursday. E.G. (2012) [6], Stroud. K.A. (2007) [8], Voyevodi N.V.V. (1980) [7] and Stephenson. (1999) [9].

1.1 Matrix

A matrix can be defined as a set of quantities arranged in a rectangular array of m rows and n columns.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots\ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots\ldots\ldots\ldots a_{1n} \\ a_{31} & a_{32} & a_{33} & \ldots\ldots\ldots\ldots a_{3n} \\ a_{m1} & a_{m2} & a_{m3} & \ldots\ldots\ldots\ldots a_{mn} \end{bmatrix}$$

The individual quantities are called the elements of the matrix of rows and columns is said to be of order m xn.

The Matrix is denoted as $\lfloor a_{ij} \rfloor_{mn}$, Where i denote number of row and j denote number of column of mn order matrix.

1.2 Determinant Matrix

The determinant of a matrix is a scalar (constant) obtained from the matrix by an appropriate evaluation depending on the order of the matrix. The determinant of matrix A is denoted by|A |.

1.3 Non - Singular Matrix

This is a square matrix whose determinant is not zero.

1.4 Adjoint Matrix

This is the transpose of the cofactors matrix. The adjoint of a matrix A is denoted by (AdjA).

1.5 The Inverse Matrix

The inverse of a matrix is another matrix which multiplies the original matrix to give an identity matrix. The inverse of a matrix A is denoted by $|A|$. .

2. The Derivation of the Determinant and the Inverse of Non – Singular 2 X 2 And 3 X 3 Matrices Using Adjoint Method

2.1 For 2 X 2 Matrix

Let us consider the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

2.1.1. The Determinant

$|A|$=det A=ad-bc$\neq 0$, Matrix A is nonsingular and its inverse exists.

If $|A|$=det A=ad-bc =0, Matrix A is singular and inverse of matrix does not exist.

2.1.2 The Adjoint

For 2 X 2 Matrix, interchange the elements of diagonal elements and change the sign of remaining elements.

$$\textbf{Adj A=}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2.1.3 The Inverse

$$A^{-1} = \frac{AdjA}{|A|}$$

$$= \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc},$$

By substituting the value of det A and Adj A, we can get the value of Inverse A

2.2 For 3 X 3 Matrix: Let us consider the matrix

$$\textbf{B=}\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

2.2.1 The Determinant

$$|B| = \det B = a_1\begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1\begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + c_1\begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2) =$ M (suppose)

$\neq 0$

As det B = M is nonsingular matrix, hence its inverse exists.

2.2.2 The Adjoint

Adjoint of 3 X 3 Matrix is transpose matrix of matrix formed by cofactor matrix.

Cofactor of $a_1 = A_{11} = \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} = b_2c_3 - b_3c_2$

Cofactor of $b_1 = A_{12} = -\begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} = -(a_2c_3 - a_3c_2)$

Cofactor of $c_1 = A_{13} = \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} = a_2b_3 - a_3b_2$

Cofactor of $a_2 = A_{21} = -\begin{bmatrix} b_1 & c_1 \\ b_3 & c_3 \end{bmatrix} = -(b_1c_3 - b_3c_1)$

Cofactor of $b_2 = A_{22} = \begin{bmatrix} a_1 & c_1 \\ a_3 & c_3 \end{bmatrix} = a_1c_3 - a_3c_1$

Cofactor of $c_2 = A_{23} = -\begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} = -(a_2b_3 - a_3b_2)$

Cofactor of $a_3 = A_{31} = \begin{bmatrix} b_1 & c_1 \\ b_2 & c_2 \end{bmatrix} = b_1c_2 - b_2c_1$

Cofactor of $b_3 = A_{32} = -\begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix} = -(a_1c_2 - a_2c_1)$

Cofactor of $c_3 = A_{33} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = -(a_1b_2 - a_2b_2)$

Adj B = $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{32} \end{bmatrix}$ **= N (suppose)**

By substituting all the values of cofactor, we will get Adj B named as N matrix.

2.2.3 The Inverse

$$B^{-1} = \frac{AdjB}{|B|} = \frac{N}{M}$$

$$= \frac{\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{32} \end{bmatrix}}{a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)}$$

Which gives the value of inverse of B.

3 The Computation of Determinant, Adjoint and Inverse Of 2 X 2 Matrices And 3 X 3 Matrices

1.3.1 Problems of 2 X 2 Matrices

Some of the problems and results of 2 x 2 matrices are shown in table 1 below.

Table 1

Problem	Matrix	Determinant	Adjoint	Inverse
1	$\begin{bmatrix} -3 & 2 \\ -1 & 7 \end{bmatrix}$	**-19**	$\begin{bmatrix} 7 & -2 \\ 1 & -3 \end{bmatrix}$	$\frac{\begin{bmatrix} 7 & -2 \\ 1 & -3 \end{bmatrix}}{-19}$
2	$\begin{bmatrix} -2 & -6 \\ 3 & 5 \end{bmatrix}$	**8**	$\begin{bmatrix} 5 & 6 \\ -3 & -2 \end{bmatrix}$	$\frac{\begin{bmatrix} 5 & 6 \\ -3 & -2 \end{bmatrix}}{8}$
3	$\begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$	**5**	$\begin{bmatrix} -4 & -3 \\ -1 & -2 \end{bmatrix}$	$\frac{\begin{bmatrix} -4 & -3 \\ -1 & -2 \end{bmatrix}}{5}$

3.2 Problems of 3 X 3 Matrices

Some of the problems and results of 3 x 3 matrices are shown in table 2 below.

Table 2

Problem	Matrix	Determinant	Adjoint	Inverse
1	$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$	**-2**	$\begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}$	$\dfrac{\begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}}{-2}$
2	$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$	**1**	$\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	$\dfrac{\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}{1}$

4 Conclusion

From above Tables 1 and 2, we can conclude that inverse of matrices of order 2 and order 3 can be easily solvable and results are accurate. With the help of these method we can easily solve the higher order matrices.

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