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Bi-Star V_4 Cordial graphs

L Pandiselvi, S Navaneethakrishnan, A Nellai Murugan

PG and Research Department of Mathematics, V. O. Chidambaram College, Tuticorin, Tamil Nadu, India.

Abstract

Let $< A, \; *>$ be any abelian group. A graph G = (V(G), E(G)) is said to be A-cordial if there is a mapping f: V(G) →A which satisfies the following two conditions with each edge
e= uv is labeled as f(u)*f(v).
(i)$\left|v_f(a) - v_f(b)\right| \leq 1$, ∀ a,b∈A
(ii)$\left|e_f(a) - e_f(b)\right| \leq 1$, ∀ a,b∈A

Where $v_f(a)$= the number of vertices with label a.p
$v_f(b)$= the number of vertices with label b.
$e_f(a)$= the number of edges with label a.
$e_f(b)$= the number of edges with label b.

We note that if A = < V_4, *> is a multiplicative group. Then the labeling is known as V_4 Cordial Labeling. A graph is called a V_4 Cordial graph if it admits a V_4- Cordial Labeling. In this paper, we proved that $B_{m,n}$ (n>2) and $B_{m,2}$ are V_4- Cordial graphs.

AMS Mathematics subject classification 2010: 05C78

Keywords: Cordial labeling, V_4 Cordial Labeling and V_4 Cordial Graph

Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For graph theoretic terminology, we referred Harary [4]. For labeling of graphs, we referred Gallian [1].

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from one another by symbols such as $v_1, v_2,...,v_n$. In a labeling of a particular type, the vertices are assigned values from a given set, the edged have a prescribed induced labeling must satisfy certain properties. The concept of graceful labeling was introduced by Rosa [3] in 1967 and subsequently by Golomb [2].

2. Preliminaries

Definition 2.1

Let G = (V, E) be a simple graph. Let f: V(G)→{0,1} and for each edge uv, assign the label $|f(u) - f(v)|$. f is called a **cordial labeling** if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called **Cordial** if it has a cordial labeling.

Definition 2.2

Let $< A, \; *>$ be any abelian group. A graph G = (V(G), E(G)) is said to be A-cordial if there is a mapping f: V(G) →A which satisfies the following two conditions with each edge
e= uv is labeled as f(u)*f(v).
(i)$\left|v_f(a) - v_f(b)\right| \leq 1$, ∀ a,b∈A
(ii)$\left|e_f(a) - e_f(b)\right| \leq 1$, ∀ a,b∈A

Where $v_f(a)$= the number of vertices with label a.
$v_f(b)$= the number of vertices with label b.
$e_f(a)$= the number of edges with label a.
$e_f(b)$= the number of edges with label b.

We note that if A = <V_4, *> is a multiplicative group. Then the labeling is known as V_4 Cordial Labeling. A graph is called a V_4 Cordial graph if it admits a V_4- Cordial Labeling.

Definition 2.3

The Bistar $B_{m,n}$ is a graph obtained from K_2 by identifying the centers of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively.

3. Main Results

Theorem: 3.1

$B_{m,n}$ is a V_4 Cordial graph, when n>2.

Proof: Let V_4= { 1,−1, i, −i}.
Let V($B_{m,n}$) ={u, v, u_i: $1 \leq i \leq m, v_i$: $1 \leq i \leq n$}.
Let E ($B_{m,n}$)={uv}∪{(uu_i):$1 \leq i \leq m$} ∪{(vv_i): $1 \leq i \leq n$}.

Define f: V($B_{m,n}$) → V_4

Case (i)
When m ≡ 0(mod 4)
The vertex labeling are,
Let f(u)=1, f(v)=1

$$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 0 (\text{mod } 4) \\ -1 & \text{if } i \equiv 1 (\text{mod } 4), \\ i & \text{if } i \equiv 2 (\text{mod } 4) \\ -i & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 1 \le i \le m$$

$f(v_1)=-1$, $f(v_2)= i$ and $f(v_3)=-i$,

$$f(v_i) = \begin{cases} -i & \text{if } i \equiv 0 (\text{mod } 4) \\ i & \text{if } i \equiv 1 (\text{mod } 4), \\ -1 & \text{if } i \equiv 2 (\text{mod } 4) \\ 1 & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 4 \le i \le n$$

The edge labeling are,

Let f(uv) = 1

$$f(uu_i) = \begin{cases} 1 & \text{if } i \equiv 0 (\text{mod } 4) \\ -1 & \text{if } i \equiv 1 (\text{mod } 4), \\ i & \text{if } i \equiv 2 (\text{mod } 4) \\ -i & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 1 \le i \le m$$

$f(vv_1)=-1$, $f(vv_2)= i$ and $f(vv_3)=-i$,

$$f(vv_i) = \begin{cases} -i & \text{if } i \equiv 0 (\text{mod } 4) \\ i & \text{if } i \equiv 1 (\text{mod } 4), \\ -1 & \text{if } i \equiv 2 (\text{mod } 4) \\ 1 & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 4 \le i \le n$$

Vertex Conditions

(i) $v_f(1)=v_f(-i) =[\frac{m+n}{4}]+1$ and $v_f(i) = v_f(-1) = [\frac{m+n}{4}]$, When n ≡ 0(mod 4)

(ii) $v_f(1) = v_f(i) = v_f(-i) =[\frac{m+n}{4}]+1$ and $v_f(-1) = [\frac{m+n}{4}]$, When n ≡ 1(mod 4)

(iii) $v_f(1)= v_f(i) = v_f(-i) =v_f(-1) = [\frac{m+n}{4}]+1$, When n ≡ 2 (mod 4)

(v)$v_f(1)= [\frac{m+n}{4}]+2$ and $v_f(i) = v_f(-i) = v_f(-1) = [\frac{m+n}{4}]+1$, When n ≡ 3 (mod 4)

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4

Edge Conditions

(i)$e_f(1) = e_f(i) = e_f(-1) = [\frac{m+n}{4}]$ and $e_f(-i) = [\frac{m+n}{4}]+1$, when n ≡ 0 (mod 4)

(ii)$e_f(1) = e_f(-1) = [\frac{m+n}{4}]$ and $e_f(-i) = e_f(i) = [\frac{m+n}{4}]+1$, when n ≡ 1 (mod 4)

(iii)$e_f(1) = [\frac{m+n}{4}]$ and $e_f(-i) = e_f(-1) = e_f(i) = [\frac{m+n}{4}]+1$, when n ≡ 2 (mod 4)

(iv) $e_f(1)= e_f(-i) = e_f(i) = e_f(-1) = [\frac{m+n}{4}]+1$, when n ≡ 3 (mod 4)

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4

Hence, $B_{m,n}$ is a V_4-Cordial Graph.

For example, the V_4-Cordial Labeling of, $B_{8,8}$, $B_{8,9}$, $B_{8,10}$, and $B_{8,11}$ is shown in below figure 3.21-3.24.

When n ≡ 0 (mod 4)

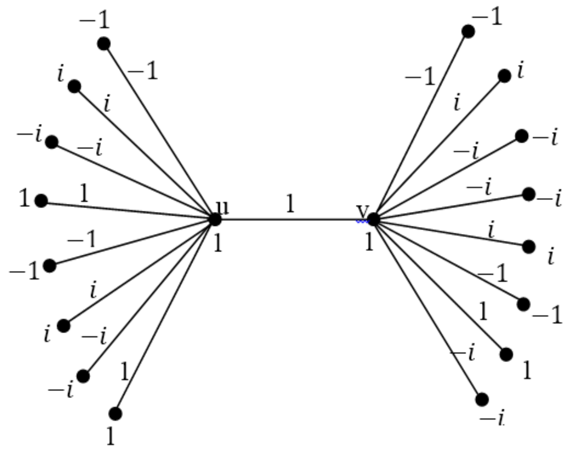

Fig 3.21

When n ≡ 1 (mod 4)

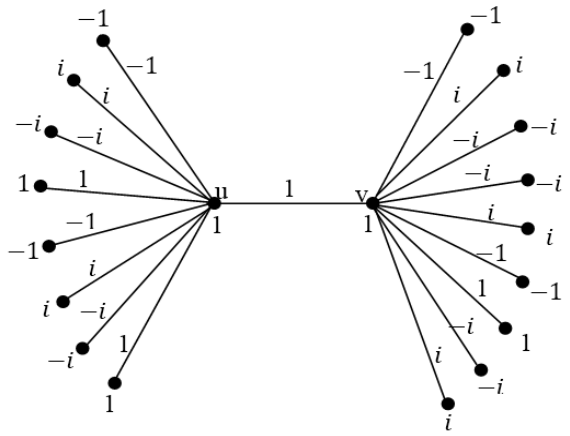

Fig 3.22

When n ≡ 2 (mod 4)

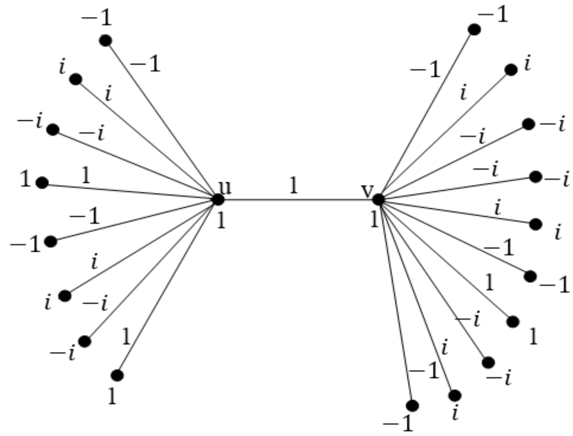

Fig 3.23

When n ≡ 3 (mod 4)

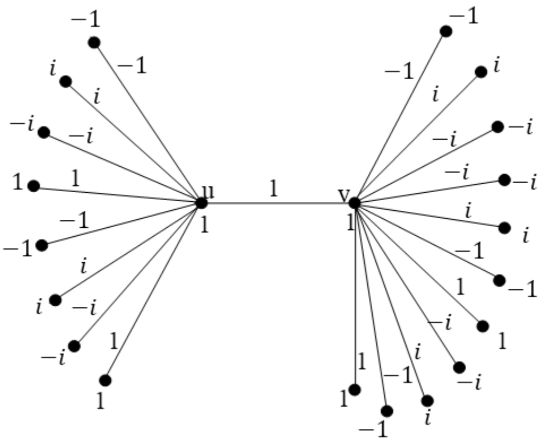

Fig 3.24

Case (ii)

When m ≡ 1(mod 4)

The vertex labeling are,

Let f(u)=1, f(v)=1

$$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod 4 \\ -1 & \text{if } i \equiv 1 \pmod 4, \\ i & \text{if } i \equiv 2 \pmod 4 \\ -i & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 1 \le i \le m$$

f(v_1)=-1, f(v_2)= i and f(v_3)=$-i$,

$$f(v_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod 4 \\ i & \text{if } i \equiv 1 \pmod 4, \\ 1 & \text{if } i \equiv 2 \pmod 4 \\ -1 & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 4 \le i \le n$$

The edge labeling are,

Let f(uv) = 1

$$f(uu_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod 4 \\ -1 & \text{if } i \equiv 1 \pmod 4, \\ i & \text{if } i \equiv 2 \pmod 4 \\ -i & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 1 \le i \le m$$

f(vv_1)=-1, f(vv_2)= i and f(vv_3)=$-i$,

$$f(vv_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod 4 \\ i & \text{if } i \equiv 1 \pmod 4, \\ 1 & \text{if } i \equiv 2 \pmod 4 \\ -1 & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 4 \le i \le n$$

Vertex Conditions

(i) $v_f(1)$=$v_f(-i)$ =$[\frac{m+n}{4}]$+1 and $v_f(i) = v_f(-1)$ = $[\frac{m+n}{4}]$,
When n ≡ 0(mod 4)
(ii) $v_f(1) = v_f(i)$ = $v_f(-i)$ = $v_f(-1)$= $[\frac{m+n}{4}]$+1,
When n ≡ 1(mod 4)
(iii) $v_f(1)$= $[\frac{m+n}{4}]$+2 and $v_f(i) = v_f(-i)$ =$v_f(-1)$ = $[\frac{m+n}{4}]$+1,
When n ≡ 2 (mod 4)
(iv)$v_f(1)$= $v_f(-1)$ =$[\frac{m+n}{4}]$+1 and $v_f(i) = v_f(-i)$ = $[\frac{m+n}{4}]$,
When n ≡ 3 (mod 4)
Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4

Edge Conditions

(i)$e_f(1)$ = $e_f(i)$ = $[\frac{m+n}{4}]$ and $e_f(-1)$ = $e_f(-i)$ = $[\frac{m+n}{4}]$+1,
When n ≡ 0 (mod 4)
(ii)$e_f(1)$ = $[\frac{m+n}{4}]$ and $e_f(-1)$ = $e_f(-i)$ = $e_f(i)$ = $[\frac{m+n}{4}]$+1,
When n ≡ 1 (mod 4)
(iii) $e_f(1)$= $e_f(-i)$ = $e_f(i)$ = $e_f(-1)$ = $[\frac{m+n}{4}]$+1,
When n ≡ 2 (mod 4)
(iv) $e_f(1)$= $e_f(-i)$ = $e_f(i)$ = $[\frac{m+n}{4}]$ and $e_f(-1)$ = $[\frac{m+n}{4}]$+1,
When n ≡ 3 (mod 4)
Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4

Hence, $B_{m,n}$ is a V_4 Cordial Graph.

For example, the V_4 Cordial Labeling of $B_{9,8}$, $B_{9,9}$, $B_{9,10}$ and $B_{9,11}$ is shown in below figure 3.25-3.28.

When n ≡ 0 (mod 4)

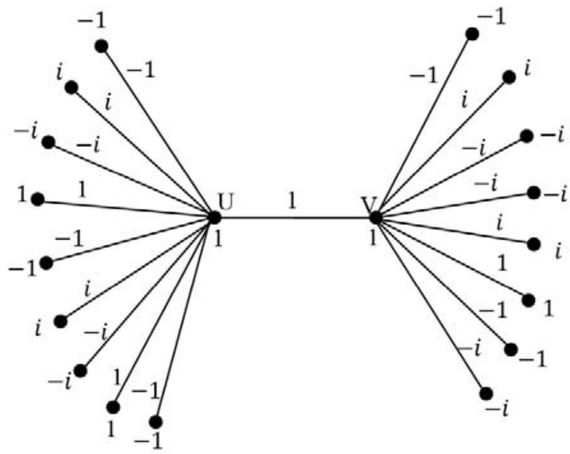

Fig 3.25

When n ≡ 1 (mod 4)

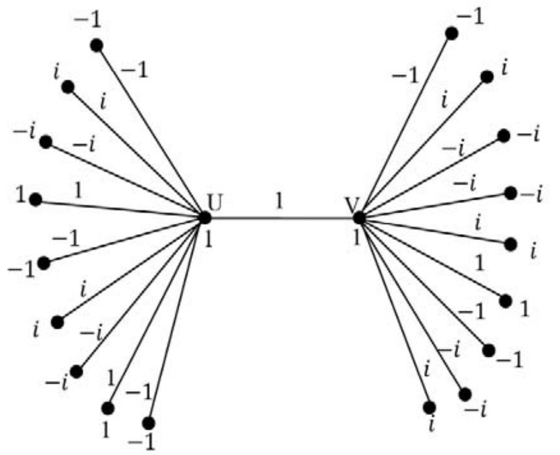

Fig 3.26

When n ≡ 2 (mod 4)

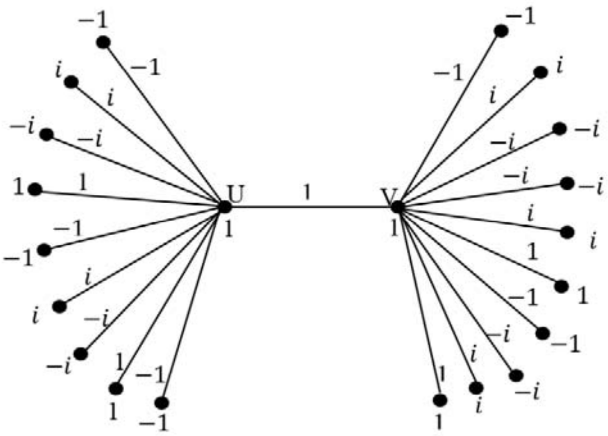

Fig 3.27

When n ≡ 3 (mod 4)

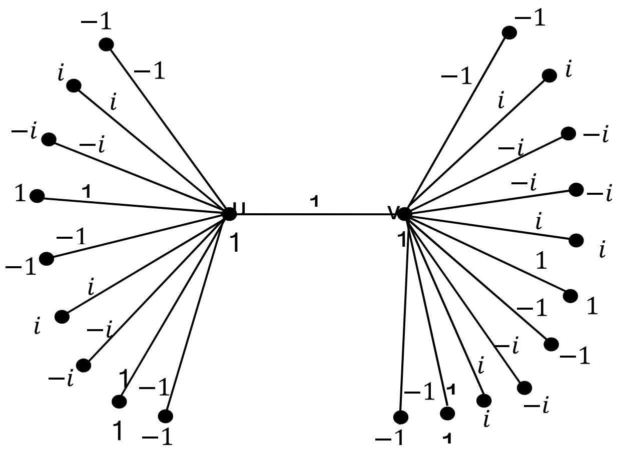

Fig 3.28

Case (iii)

When m ≡ 2(mod 4)

The vertex labeling are,

Let f(u)=1, f(v)=1

$$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 0 (\text{mod } 4) \\ -1 & \text{if } i \equiv 1 (\text{mod } 4), \\ i & \text{if } i \equiv 2 (\text{mod } 4) \\ -i & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 1 \le i \le m$$

$f(v_1)=-1$, $f(v_2)= i$ and $f(v_3)=-i$,

$$f(v_i) = \begin{cases} -i & \text{if } i \equiv 0 (\text{mod } 4) \\ 1 & \text{if } i \equiv 1 (\text{mod } 4), \\ -1 & \text{if } i \equiv 2 (\text{mod } 4) \\ i & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 4 \le i \le n$$

The edge labeling are,

Let f(uv) = 1

$$f(uu_i) = \begin{cases} 1 & \text{if } i \equiv 0 (\text{mod } 4) \\ -1 & \text{if } i \equiv 1 (\text{mod } 4), \\ i & \text{if } i \equiv 2 (\text{mod } 4) \\ -i & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 1 \le i \le m$$

$f(vv_1)=-1$, $f(vv_2)= i$ and $f(vv_3)=-i$,

$$f(vv_i) = \begin{cases} -i & \text{if } i \equiv 0 (\text{mod } 4) \\ 1 & \text{if } i \equiv 1 (\text{mod } 4), \\ -1 & \text{if } i \equiv 2 (\text{mod } 4) \\ i & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}, \quad 4 \le i \le n$$

Vertex Conditions

(i) $v_f(1) = v_f(i) = v_f(-i) = v_f(-1) = [\frac{m+n}{4}]+1$,

When n ≡ 0(mod 4)

(ii) $v_f(1) = [\frac{m+n}{4}]+2$ and $v_f(i) = v_f(-i) = v_f(-1) = [\frac{m+n}{4}]+1$,

When n ≡ 1 (mod 4)

(iii) $v_f(1) = v_f(-1) = [\frac{m+n}{4}]+1$ and $v_f(i) = v_f(-i) = [\frac{m+n}{4}]$,

when n ≡ 2 (mod 4)

(iv) $v_f(1) = v_f(-1) = v_f(i) = [\frac{m+n}{4}]+1$ and $v_f(-i) = [\frac{m+n}{4}]$,

When n ≡ 3 (mod 4)

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4

Edge Conditions

(i)$e_f(1) = [\frac{m+n}{4}]$ and $e_f(-1) = e_f(-i) = e_f(i) = [\frac{m+n}{4}]+1$,

When n ≡ 0 (mod 4)

(ii) $e_f(1) = e_f(-i) = e_f(i) = e_f(-1) = [\frac{m+n}{4}]+1$,

When n ≡ 1 (mod 4)

(iii) $e_f(1) = e_f(-i) = e_f(i) = [\frac{m+n}{4}]$ and $e_f(-1) = [\frac{m+n}{4}]+1$,

Wwhen n ≡ 2 (mod 4)

(iv)$e_f(1) = e_f(-i) = [\frac{m+n}{4}]$ and $e_f(-1) = e_f(i) = [\frac{m+n}{4}]+1$,

When n ≡ 3 (mod 4)

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4

Hence, $B_{m,n}$ is a V_4 Cordial Graph.

For example, the V_4 Cordial Labeling of $B_{6,8}$, $B_{6,9}$, $B_{6,10}$ and $B_{6,11}$ is shown in below figure 3.29-3.32.

When n ≡ 0 (mod 4)

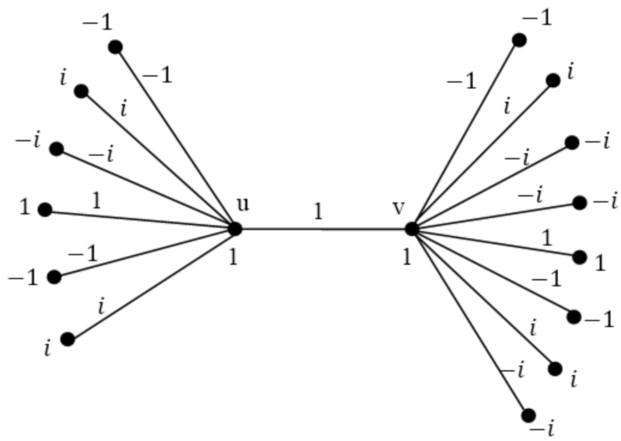

Fig 3.29

When n ≡ 1 (mod 4)

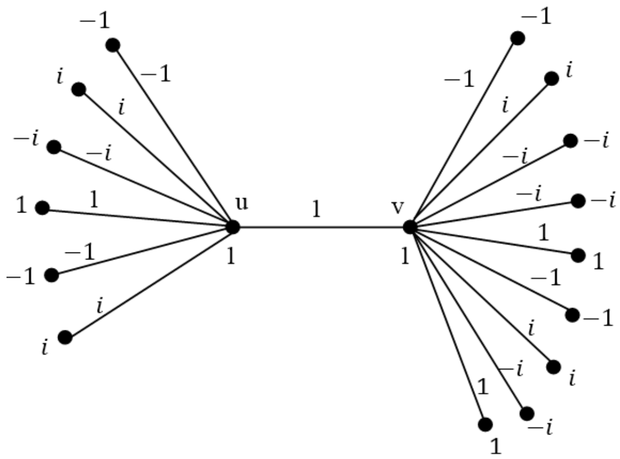

Fig 3.30

When n ≡ 2 (mod 4)

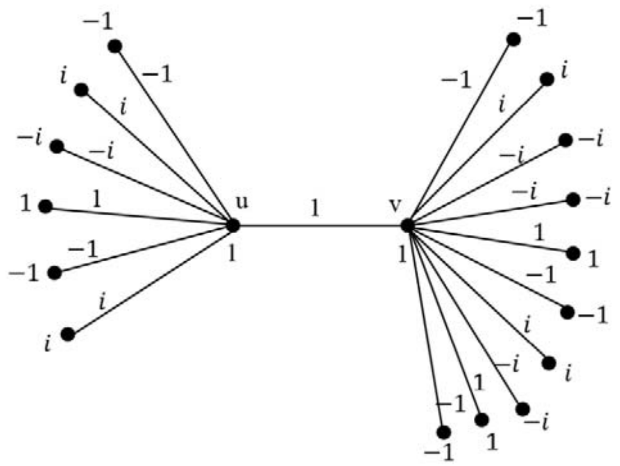

Fig 3.31

When n ≡ 3 (mod 4)

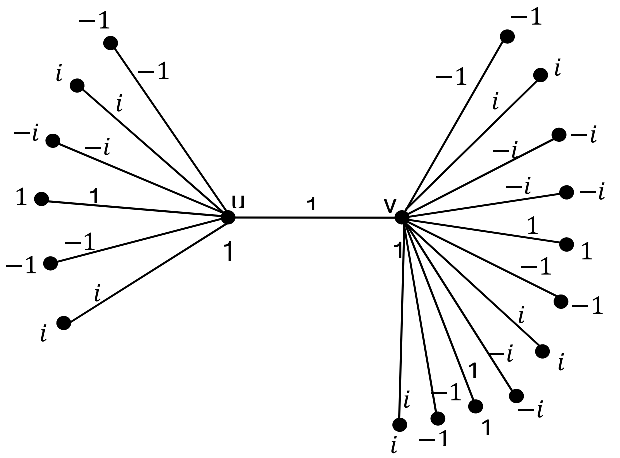

Fig 3.32

Case (iv)
When m ≡ 3(mod 4)

The vertex labeling are,

Let f(u)=1, f(v)=1

$$f(u_i) = \begin{cases} 1 & \text{if i} \equiv 0 \text{(mod 4)} \\ -1 & \text{if i} \equiv 1 \text{(mod 4),} \\ i & \text{if i} \equiv 2 \text{(mod 4)} \\ -i & \text{if i} \equiv 3 \text{(mod 4)} \end{cases}, \quad 1 \le i \le m$$

f(v_1)=−1, f(v_2)= i and f(v_3)=$-i$,

$$f(v_i) = \begin{cases} 1 & \text{if i} \equiv 0 \text{(mod 4)} \\ i & \text{if i} \equiv 1 \text{(mod 4),} \\ -i & \text{if i} \equiv 2 \text{(mod 4)} \\ -1 & \text{if i} \equiv 3 \text{(mod 4)} \end{cases}, \quad 4 \le i \le n$$

The edge labeling are,

Let f(uv) = 1

$$f(uu_i) = \begin{cases} 1 & \text{if i} \equiv 0 \text{(mod 4)} \\ -1 & \text{if i} \equiv 1 \text{(mod 4),} \\ i & \text{if i} \equiv 2 \text{(mod 4)} \\ -i & \text{if i} \equiv 3 \text{(mod 4)} \end{cases}, \quad 1 \le i \le m$$

f(vv_1)=−1, f(vv_2)= i and f(vv_3)=$-i$,

$$f(vv_i) = \begin{cases} 1 & \text{if i} \equiv 0 \text{(mod 4)} \\ i & \text{if i} \equiv 1 \text{(mod 4),} \\ -i & \text{if i} \equiv 2 \text{(mod 4)} \\ -1 & \text{if i} \equiv 3 \text{(mod 4)} \end{cases}, \quad 4 \le i \le n$$

Vertex Conditions

(i) $v_f(1)$= [$\frac{m+n}{4}$]+2 and $v_f(i) = v_f(-i)$ =$v_f(-1)$ = [$\frac{m+n}{4}$]+1,
When n ≡ 0 (mod 4)
(ii)$v_f(1)$= $v_f(i)$ =[$\frac{m+n}{4}$]+1 and $v_f(-1) = v_f(-i)$ = [$\frac{m+n}{4}$],
When n ≡ 1 (mod 4)
(iii)$v_f(1)$= $v_f(-i)$ =$v_f(i)$ =[$\frac{m+n}{4}$]+1 and $v_f(-1)$ = [$\frac{m+n}{4}$],
When n ≡ 2 (mod 4)
(iv) $v_f(1) = v_f(i)$ = $v_f(-i)$ = $v_f(-1)$= [$\frac{m+n}{4}$]+1, when n ≡ 3 (mod 4)
Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4

Edge Conditions

(i) $e_f(1)$= $e_f(-i)$ = $e_f(i)$ = $e_f(-1)$ = [$\frac{m+n}{4}$]+1,
When n ≡ 0 (mod 4)
(ii) $e_f(1)$= $e_f(-i)$ = $e_f(-1)$ = [$\frac{m+n}{4}$] and $e_f(i)$ = [$\frac{m+n}{4}$]+1,
\When n ≡ 1 (mod 4)
(iii)$e_f(1)$ = $e_f(-1)$ = [$\frac{m+n}{4}$] and $e_f(-i) = e_f(i)$ = [$\frac{m+n}{4}$]+1,
When n ≡ 2 (mod 4)
(iv)$e_f(1)$ = [$\frac{m+n}{4}$] and $e_f(-1)$ = $e_f(-i)$ = $e_f(i)$ = [$\frac{m+n}{4}$]+1,
When n ≡ 3 (mod 4)
Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4

Hence, $B_{m,n}$ is a V_4 Cordial Graph.

For example, the V_4 Cordial Labeling of $B_{7,8}, B_{7,9}, B_{7,10}$ and $B_{7,11}$ is shown in below figure 3.33-3.36.

When n ≡ 0 (mod 4)

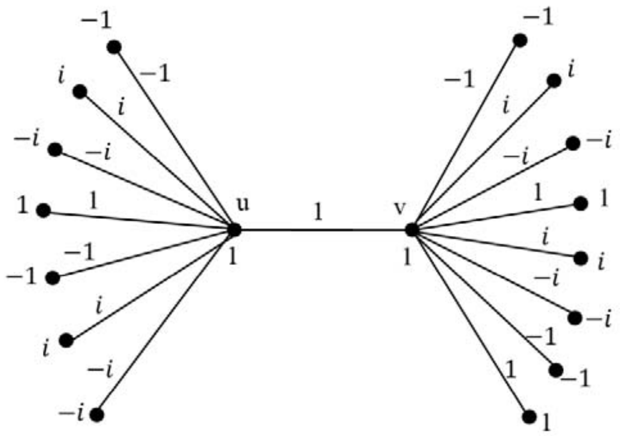

Fig 3.33

When n ≡ 1 (mod 4)

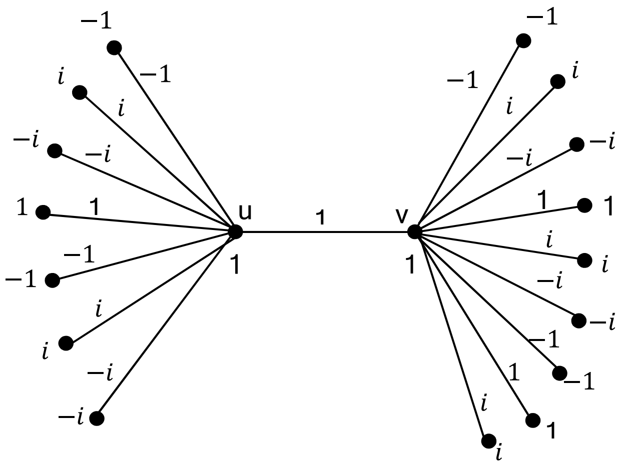

Fig 3.34

When n ≡ 2 (mod 4)

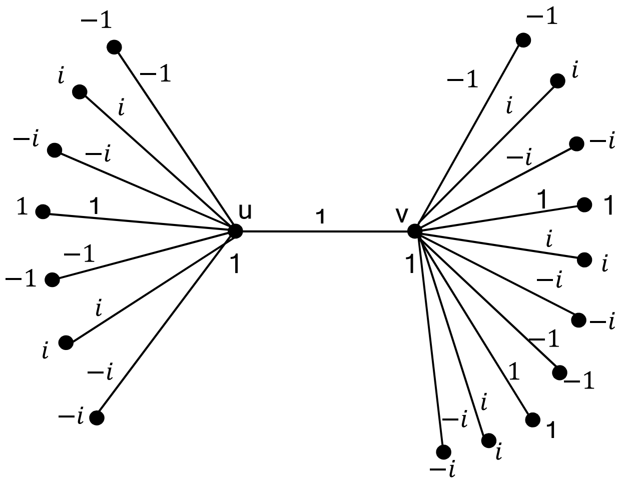

Fig 3.35

When n ≡ 3 (mod 4)

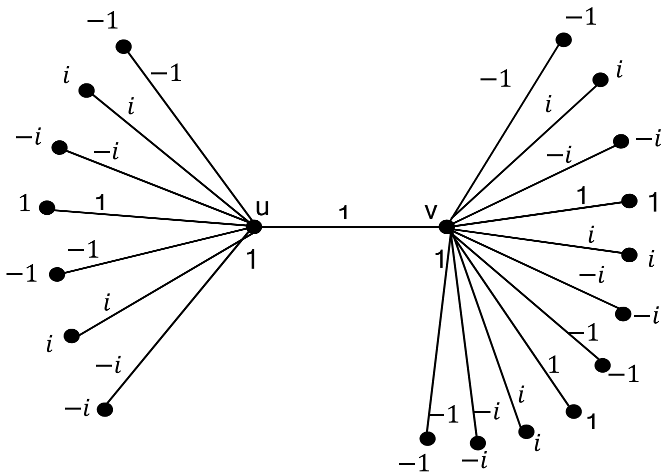

Fig 3.36

Theorem: 3.4

$B_{m,2}$ is a V_4- Cordial graph.

Proof: Let V_4= { 1,−1, i, −i}.

Let V($B_{m,n}$) ={u, v, u_i: $1 \le i \le m, v_i$: $1 \le i \le n$}.

Let E ($B_{m,n}$) ={uv}∪{(uu_i): $1 \le i \le m$}∪{(vv_i): $1 \le i \le n$}.

Define f: V($B_{m,n}$) → V_4

Case (i)
When m ≡ 0(mod 4)
The vertex labeling are,

Let f(u)=1, f(v)=1, f(v_1)= i and f(v_2)= $-i$

$$f(u_i) = \begin{cases} 1 & \text{if i} \equiv 0(\text{mod } 4) \\ -1 & \text{if i} \equiv 1(\text{mod } 4), \\ i & \text{if i} \equiv 2(\text{mod } 4) \\ -i & \text{if i} \equiv 3(\text{mod } 4) \end{cases}, \ 1 \le i \le m$$

The edge labeling are,

Let f(uv)=−1, f(vv_1)= $-i$ and f(vv_2)=i,

$$f(uu_i) = \begin{cases} 1 & \text{if i} \equiv 0(\text{mod } 4) \\ -1 & \text{if i} \equiv 1(\text{mod } 4), \\ i & \text{if i} \equiv 2(\text{mod } 4) \\ -i & \text{if i} \equiv 3(\text{mod } 4) \end{cases}, \ 1 \le i \le m$$

Vertex Conditions

(i) $v_f(1)$= $v_f(i) = v_f(-i)$ =$v_f(-1)$ = $\frac{m+n+2}{4}$,

Hence, it satisfies the condition of $\left|v_f(a) - v_f(b)\right| \le 1$, ∀ a,b∈ V_4

Edge Conditions

(i) $e_f(1)$= $\frac{m+n-2}{4}$ and $e_f(-i)$ = $e_f(i)$ = $e_f(-1)$ = $\frac{m+n+2}{4}$, Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \leq 1$, ∀ a,b∈ V_4

Hence, $B_{m,2}$ is a V_4 Cordial Graph.

For example, the V_4 Cordial Labeling of, $B_{8,2}$ is shown in below figure 3.51

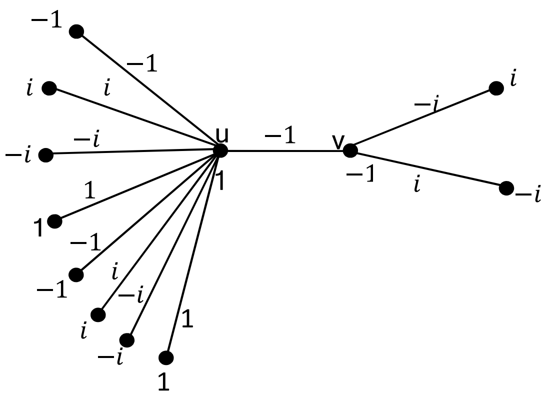

Fig 3.51

Case (ii)
When m ≡ 1(mod 4)

The vertex labeling are,

Let f(u)=1, f(v)= i, f(v_1)= $-i$ and f(v_2)= -1

$$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod 4 \\ -1 & \text{if } i \equiv 1 \pmod 4, \\ i & \text{if } i \equiv 2 \pmod 4 \\ -i & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 1 \leq i \leq m$$

The edge labeling are,

Let f(uv)= i, f(vv_1)= 1 and f(vv_2)= $-i$,

$$f(uu_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod 4 \\ -1 & \text{if } i \equiv 1 \pmod 4, \\ i & \text{if } i \equiv 2 \pmod 4 \\ -i & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 1 \leq i \leq m$$

Vertex Conditions

(i) $v_f(1)= v_f(i) = v_f(-i) = \frac{m+n+1}{4}$ $v_f(-1) = \frac{m+n+1}{4}+1$,

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \leq 1$, ∀ a,b∈ V_4

Edge Conditions

(i) $e_f(1)= e_f(-i) = e_f(i) = e_f(-1) = \frac{m+n+1}{4}$,

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \leq 1$, ∀ a,b∈ V_4

Hence, $B_{m,2}$ is a V_4 Cordial Graph.

For example, the V_4 Cordial Labeling of, $B_{5,2}$ is shown in below figure 3.52.

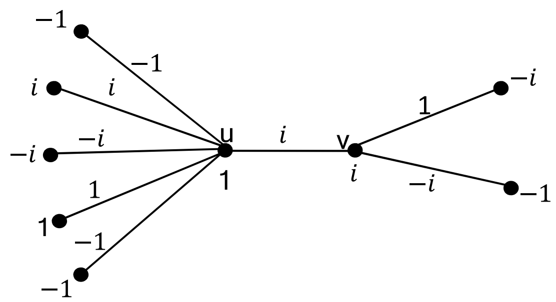

Fig 3.52

Case (iii)
When m ≡ 2(mod 4)

The vertex labeling are,

Let f(u)=1, f(v)= $-i$, f(v_1)= 1 and f(v_2)= i

$$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod 4 \\ -1 & \text{if } i \equiv 1 \pmod 4, \\ i & \text{if } i \equiv 2 \pmod 4 \\ -i & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 1 \leq i \leq m$$

The edge labeling are,

Let f(uv)= i, f(vv_1)= $-i$ and f(vv_2)= 1,

$$f(uu_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod 4 \\ -1 & \text{if } i \equiv 1 \pmod 4, \\ i & \text{if } i \equiv 2 \pmod 4 \\ -i & \text{if } i \equiv 3 \pmod 4 \end{cases}, \quad 1 \leq i \leq m$$

Vertex Conditions

(i) $v_f(1)= v_f(i) = \frac{m+n}{4} + 1$ *and* $v_f(-i) = v_f(-1) = \frac{m+n}{4}$,

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \leq 1$, ∀ a,b∈ V_4

Edge Conditions

(i) $e_f(1)= e_f(i) = e_f(-1) = \frac{m+n}{4}$ and $e_f(-i) = \frac{m+n}{4}+1$

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \leq 1$, ∀ a,b∈ V_4

Hence, $B_{m,n}$ is a V_4 Cordial Graph.

For example, the V_4 Cordial Labeling of $B_{6,2}$ is shown in below figure 3.53.

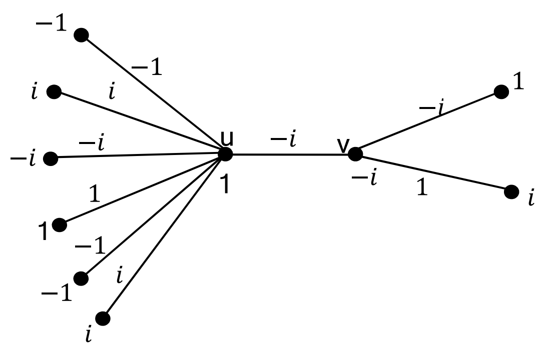

Fig 3.53

Capse (iv)
When m ≡ 3(mod 4)
The vertex labeling are,

Let f(u)=1, f(v)= 1, f(v_1)= -1 and f(v_2)= i

$$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 0(\text{mod } 4) \\ -1 & \text{if } i \equiv 1(\text{mod } 4), \\ i & \text{if } i \equiv 2(\text{mod } 4) \\ -i & \text{if } i \equiv 3(\text{mod } 4) \end{cases}, \quad 1 \le i \le m$$

The edge labeling are,

Let f(uv)= 1, f(vv_1)= -1 and f(vv_2)= i,

$$f(uu_i) = \begin{cases} 1 & \text{if } i \equiv 0(\text{mod } 4) \\ -1 & \text{if } i \equiv 1(\text{mod } 4), \\ i & \text{if } i \equiv 2(\text{mod } 4) \\ -i & \text{if } i \equiv 3(\text{mod } 4) \end{cases}, \quad 1 \le i \le m$$

Vertex Conditions

(i) $v_f(1)= v_f(-1) = v_f(i) = \frac{m+n+3}{4}$ and $v_f(-i)= \frac{m+n-1}{4}$,

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4

Edge Conditions

(i) $e_f(1)= e_f(-i)= \frac{m+n-1}{4}$ and $e_f(-1)= e_f(i) = \frac{m+n+3}{4}$

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4

Hence, $B_{m,n}$ is a V_4-Cordial Graph .

For example, the V_4-Cordial Labeling of $B_{7,2}$ is shown in below figure 3.54

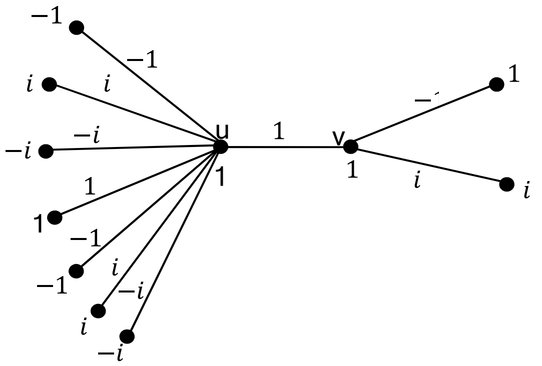

Fig 3.54

4. References

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