

## An introduction to Metamaterials

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### Abstract

This article is an introduction to the interesting field of metamaterials. It traces the history of its development and introduces the reader to the electromagnetic aspects of metamaterials.

**Keywords:** metamaterials, permittivity, permeability

### Introduction

In 1948, W.E. Kock suggested that, a di-electric lens could be made lighter by replacing heavy refractive materials with a mixture of small metal spheres in a light-weight matrix [1]. This dielectric material was defined in this pioneering work as a composite, reproducing on a much larger scale processes occurring in the molecules of a usual dielectric.

This concept probably was first suggested by Lord Rayleigh in his pioneering work [2]. To the best of our knowledge, the first attempt to explore the concept of “artificial” materials apparently traces back to the late part of the nineteenth century when in 1898 Jagadis Chander Bose conducted the first microwave experiment on twisted structures—geometries that were essentially artificial chiral elements by today’s terminology [3]. In 1914, Lindman worked on “artificial” chiral media by embedding many randomly oriented small wire helices in a host medium [4]. In 1948, Kock [5] made lightweight microwave lenses by arranging conducting spheres, disks, and strips periodically and effectively tailoring the effective refractive index of the artificial media. Since then, artificial complex materials have been the subject of research for many investigators worldwide. In recent years new concepts in synthesis and novel fabrication techniques have allowed the construction of structures and composite materials that mimic known material responses or that qualitatively have new, physically realizable response functions that do not occur or may not be readily available in nature.

The modern history of “electromagnetic metamaterials” generally referred to as ‘metamaterials’ can be counted from the seminal paper of J.B.Pendry [7] where he set an ambitious goal to create so-called perfect lens. The creation of a perfect lens necessitates the medium having negative permittivity and permeability in the same frequency range.

Metamaterials, though a perfect definition is still to be coined, are artificial structures designed to have properties not available in nature. They resemble natural crystals as they are built from periodically arranged unit cells, each with a side length of ‘ $a$ ’. The unit cells are not made of physical atoms or molecules but, instead, contain small metallic resonators which interact with an external electromagnetic wave that has a wavelength  $\lambda$ . The manner in which the incident light wave

interacts with these metallic “meta-atoms” of a metamaterial determines the medium’s electromagnetic properties – which may, hence, be made to enter highly unusual regimes, such as one where the electric permittivity and the magnetic permeability become simultaneously (in the same frequency region) negative. Metamaterials, thus are artificial structures designed to have properties not available in nature.

The response of a metamaterial to an incident electromagnetic wave can be classified by ascribing to it an effective (averaged over the volume of a unit cell) permittivity

$\epsilon_{eff} = \epsilon_0 \epsilon_r$ , and effective permeability  $\mu_{eff} = \mu_0 \mu_r$ . In order to introduce such a description, one requires that the size of the artificial resonators characterized by ‘ $a$ ’ be much smaller than the wavelength  $\lambda$ , i.e.  $a \ll \lambda$ . As long as this criterion is fulfilled, one may normally assume that the response of the

medium is local, i.e. that the value of  $\epsilon_{eff}$  and  $\mu_{eff}$  averaged over a given unit cell do not depend on the wave-vector nor on the corresponding values of these parameters at neighbouring unit cells. Hence, in such a medium, the effects of spatial dispersion may legitimately be ignored.

Of late, the idea of complex materials which possesses negative real values of permittivity and permeability at certain frequencies has received considerable attention. In 1967, Veselago [8] theoretically investigated plane-wave propagation in a material whose permittivity and permeability were assumed to be simultaneously negative [4]. His theoretical study showed that for a monochromatic uniform plane wave in such a medium the direction of the Poynting vector is anti-parallel to the direction of the phase velocity, contrary to the case of plane wave propagation in conventional simple media. In recent years, Smith, Schultz, and their group constructed such a composite medium for the microwave regime and demonstrated experimentally the presence of anomalous refraction in this medium [9].

The perfect lens described in [1] is not attainable as any other perfectness [5]. But it is an exciting task to approach this perfect image, i.e., to obtain a sub-wavelength image in farzone of a source. It has been known since the time of Lord Rayleigh that diffraction imposes a limit to the focal spot size. This limit is the main restriction in what concerns the spatial resolution of two closely positioned point sources or the imaging of small-scale details in complex sources [3]. The best possible resolution and the smallest size of the focal spot

offered by lenses is approximately equal to  $0.4\lambda$ . This result for the focal spot diameter is still considered as not sub-wavelength imaging and is allowed by the diffraction limit. The diffraction limit, however, implies that the image is created only by propagating waves, and the sub-wavelength information contained in “evanescent waves” exponentially decaying with distance from the field source, is lost in the image domain.

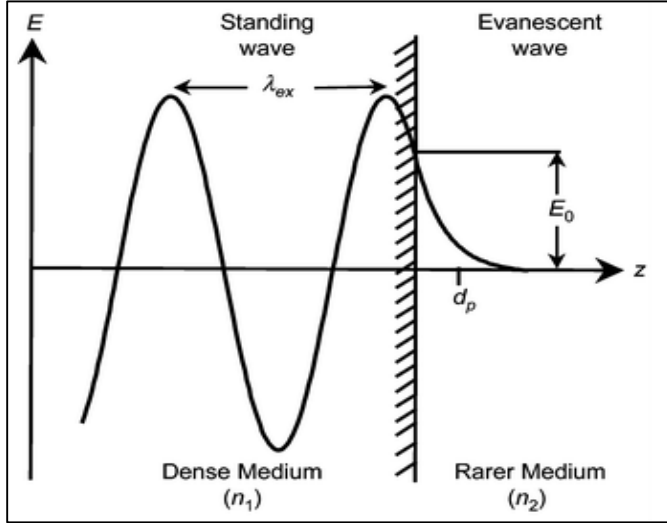


Fig 1: Evanescent waves

These metamaterials can in principle be synthesized by embedding various constituents/inclusions with novel geometric shapes and forms in some host media. Various types of electromagnetic composite media, such as double-negative (DNG) materials, chiral materials, omega media, wire media, bianisotropic media, linear and nonlinear media, and local and nonlocal media, to name a few, have been studied by various research groups worldwide.

As is well known, in particulate composite the response of a system to the presence of an electromagnetic field is determined to a large extent by the properties of the materials involved. We describe these properties by defining the macroscopic parameters permittivity  $\epsilon$  and permeability  $\mu$  of these materials. This allows for the classification of a medium as follows. A medium with both permittivity and permeability greater than zero ( $\epsilon > 0, \mu > 0$ ) will be designated a double positive (DPS) medium. Most naturally occurring media (e.g., dielectrics) fall under this designation. A medium with permittivity less than zero and permeability greater than zero ( $\epsilon < 0, \mu > 0$ ) will be designated an epsilon-negative (ENG) medium. In certain frequency regimes many plasmas exhibit this characteristic. For example, noble metals (e.g., silver, gold) behave in this manner in the infrared (IR) and visible frequency domains. A medium with the permittivity greater than zero and permeability less than zero ( $\epsilon > 0, \mu < 0$ ) will be designated a munegative (MNG) medium. In certain frequency regimes some gyrotropic materials exhibit this characteristic. Artificial materials have been constructed that also have DPS, ENG, and MNG properties. A medium with both the permittivity and permeability less than zero ( $\epsilon < 0, \mu < 0$ ) will be designated a DNG medium.

ENG Material ( $\epsilon < 0, \mu > 0$ ) <i>Plasmas</i>	DPS Material ( $\epsilon > 0, \mu > 0$ ) <i>Dielectric</i>
DNG Material ( $\epsilon < 0, \mu < 0$ ) <i>Not found in Nature, But physically realizable</i>	MNG Material ( $\epsilon > 0, \mu < 0$ ) <i>Gyrotropic magnetic materials</i>

Fig 2: class of materials

**Frequency Response**

While one often describes a material by some constant (frequency independent) value of the permittivity and permeability, in reality all material properties are frequency dependent. There are several material models that have been constructed to describe the frequency response of materials. Because the magnetic field of an electromagnetic wave is smaller than its electric field by the wave impedance of the medium in which it is propagating, one generally focuses attention on how the electron motion in the presence of the nucleus and, hence, the basic dipole moment of this system are changed by the electric field. Understanding this behavior leads to a model of the electric susceptibility of the medium and, hence, its permittivity. On the other hand, there are many media for which the magnetic field response is dominant. One can generally describe the magnetic response of a material in a fashion completely dual to that of the electric field using the magnetic susceptibility and, hence, its permeability. While the magnetic dipoles physically arise from moments associated with current loops, they can be described mathematically by magnetic charge and current analogs of the electric cases. One of the most well-known material models is the Lorentz model. It is derived by a description of the electron motion in terms of a driven, damped harmonic oscillator. To simplify the discussion, we will assume that the charges are allowed to move in the same direction as the electric field. The Lorentz model then describes the temporal response of a component of the polarization field of the medium to the same component of the electric field as

$$\frac{d}{dt} P_i + \Gamma_L \frac{d}{dt} P_i + \omega_0^2 P_i = \epsilon_0 \chi_L E_i \tag{1.1}$$

The first term on the left accounts for the acceleration of the charges, the second accounts for the damping mechanisms of the system with damping coefficient  $\Gamma_L$  and the third accounts for the restoring forces with the characteristic frequency  $\omega_0 = \omega_0/2\pi$ . The driving term exhibits a coupling coefficient  $\chi_L$ . The response in the frequency domain, assuming the engineering  $\exp(+j\omega t)$  time dependence, is given by the expression

$$P_i(\omega) = \frac{\chi_L}{-\omega^2 + j\Gamma_L\omega + \omega_0^2} \varepsilon_0 E_i(\omega) \quad (1.2)$$

With small losses  $\frac{\Gamma_L}{\omega_0} \ll 1$  the response is clearly resonant at the natural frequency  $\omega_0$ . The polarization and electric fields are related to the electric susceptibility as

$$\chi_{e, \text{Lorentz}}(\omega) = \frac{P_i(\omega)}{\varepsilon_0 E_i(\omega)} = \frac{\chi_L}{-\omega^2 + j\Gamma_L\omega + \omega_0^2} \quad (1.3)$$

The permittivity is then obtained immediately as

$$\varepsilon_{\text{Lorentz}}(\omega) = \varepsilon_0 [1 + \chi_{e, \text{Lorentz}}(\omega)]$$

There are several well-known special cases of the Lorentz model. When the acceleration term is small in comparison to the others, one obtains the *Debye* model:

$$\Gamma_d \frac{d}{dt} P_i + \omega_0^2 P_i = \varepsilon_0 \chi_d E_i \quad \chi_{e, \text{Debye}}(\omega) = \frac{\chi_d}{j\Gamma_d \omega + \omega_0^2} \quad (1.4)$$

When the restoring force is negligible, one obtains the *Drude* model:

$$\frac{d}{dt} P_i + \Gamma_D \frac{d}{dt} P_i = \varepsilon_0 \chi_D E_i \quad \chi_{e, \text{Drude}}(\omega) = \frac{\chi_D}{-\omega^2 + j\Gamma_D \omega} \quad (1.5)$$

where the coupling coefficient is generally represented by the plasma frequency  $\chi_D = \omega_p^2$ . In all of these models, the high-frequency limit reduces the permittivity to that of free space. Assuming that the coupling coefficient is positive, then only the Lorentz and the Drude models can produce negative permittivities. Because the Lorentz model is resonant, the real part of the susceptibility and, hence, that of the permittivity become negative in a narrow frequency region immediately above the resonance. On the other hand, the Drude model can yield a negative real part of the permittivity over a wide

spectral range, that is, for  $\omega < \sqrt{\omega_p^2 - \Gamma_D^2}$ . Similar magnetic response models follow immediately. The corresponding magnetization field components  $M_i$  and the magnetic susceptibility  $\chi_m$  equations are obtained from the polarization and electric susceptibility expressions with the

replacements  $E_i \rightarrow H_i, \frac{P_i}{\varepsilon_0} \rightarrow M_i$ . The permeability is given as  $\mu(\omega) = \mu_0 [1 + \chi_m(\omega)]$ .

Metamaterials have necessitated the introduction of generalizations of these models. For instance, the most general second-order model that has been introduced for metamaterial studies is the two-time-derivative Lorentz metamaterial (2TDLM) model [10-12]

$$\frac{d^2}{dt^2} P_i + \Gamma_L \frac{d}{dt} P_i + \omega_0^2 P_i = \frac{\chi_a \omega_p^2 + j\chi_b \omega_p \omega - \chi_\gamma \omega^2}{-\omega^2 + j\Gamma_L \omega + \omega_0^2} E_i \quad (1.6)$$

This 2TDLM model incorporates all the standard Lorentz model behaviors including the resonance behavior at  $\omega_0$  but allows for additional driving mechanisms that are important when considering time-varying phenomena. It satisfies a generalized Kramers-Kronig relation and is causal if  $\chi_\gamma > -1$ . Frequency behavior has the peculiar property that if  $-1 < \chi_\gamma < 0$ , then  $\lim_{\omega \rightarrow \infty} \varepsilon(\omega) < 1$ , which leads to the interesting but still controversial trans-vacuum-speed (TVS) effect [13, 14].

### Conclusion

This branch of study has expanded so much that it is impossible to represent all its sub-fields and topics of research in one introductory article. As a result, the present collection of papers in this issue is by no means exhaustive. Multivolume handbooks would be necessary to include all the various areas of this field. We refer the interested readers to numerous other special issues of various technical journals, books, and monographs on metamaterials in the fields of engineering, physics, material science, chemistry, and mathematics among others. The metamaterial research is vibrant, progressing steadily and strongly into various exciting forefronts which include quantum metamaterials, superconducting metamaterials, tunable and reconfigurable metamaterials, photonic metamaterials, infra-red metamaterials, and terahertz metamaterials, just to name a few. So the future of this field is quite bright.

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