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# Cocentroidal Matrices of Class 1 and Class 3

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**Abstract**

Content of this paper reflects about class preservation property of cocentroidal matrices. We have identified square matrices of two different classes-1 and 3*, Matrices falling under these classes stand for their unique identity and special algebraic properties. The set of cocentroidal matrices to a given root matrix of any class, either1 or 3, preserves the characteristic property of the corresponding class. Graphical presentation in each case makes the notion more sharper. [* Matrices of class 1 and class 3 are the members of infinite class of square matrices for which the algebraic sum of each column entries ( case of class 1) and sum of each column and each row entries ( case of class 3) remains a real constant; we call this constant – a Libra value.]

**Keywords:** L (A), CJ1 (n×n, L (A) = p), CJ3 (n×n, L (A) = p), G*(A)

## 1. Introduction

We have classified square matrices into two sets and we, at this point, discuss about only that part of the set whose member matrices follow certain pre assigned constraints. We have two special cases of the same group of matrices;

1) Square matrices such that the algebraic sum of each column entries remains a real constant. We call this constant sum as libra value and identify this set as the matrices of class1 [1].
2) In this part, we have square matrices such that the algebraic sum of each column entries as well as each row entries remain a real constant. We call this constant sum as libra value and identify this set as the matrices of class 3 [2].

The major notion in this content is to show that there exists an infinite set of square matrices for which the centroid of their physical / solid structure remains the same as that of the original matrix—we shall call it a root matrix. In addition to that all such cocentroidal matrices derived from the root matrix preserves the class, either 1 or 3, to which the original root matrix belongs. Some important properties associated to such matrices are also mentioned at right place in this matter. We have made a justifiable attempt to show the graph of cocentroidal matrices.

The following part introduces the same concept in abstract terminology.

## 2. Previous background of Class1 and class3

### 2.1 Property 1 (P1)

Consider a square matrix A = $\left(a_{ij}\right)_{n\times n}$ on R, ∀ i = 1 to n and j = 1 to n. where n ∈ N.

If $\sum_{i=1}^{n} a_{ij}$= Constant for each j = 1, 2 ...n.

i.e. if the sum of all the entries of a column for each one of the columns of the given matrix A, remains the same real constant than the matrix is said to satisfy the property P1 [1].

### 2.2 Property 2 (P2)

Consider a square matrix, A= $\left(a_{ij}\right)_{m\times n}$on R, ∀ i = 1 to m and j = 1 to n. where m, n ∈ N

If $\sum_{i=1}^{m} a_{ij}$= Constant for each j = 1, 2….n.

and $\sum_{j=1}^{n} a_{ij}$= Constant for each i = 1, 2,….m.

I.e. If the sum of all the entries of a column and a row for each one of the columns and rows of the given matrix A, remains the same real constant than the matrix is said to satisfy the property P2 [2].

### 2.3 Libra value

Libra value of a given class of matrices is the real constant which is associated with the property of a class. Libra value of the square matrix A will be denoted by the symbol L (A)[1].

### 2.4 Class1 (CJ1)

A set of matrices which observe the property P1 constitutes class1; denoted as CJ1.

CJ1 = {A |A = $\left(a_{ij}\right)_{m\times m}$}, A satisfies P1 and L (A) = p; p ∈ R for a given matrix A[1]. (1)

We denote, for the given matrix A, the notation A ∈ CJ1 (m×m, p).

For example, A = $\begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ ∈ CJ1 (3, 4) with L (A) = 4.

### 2.5 Class 3 (CJ3)

A square matrix is said to be of the class3 if it possesses the property2.

CJ3 = {A |A = $\left(a_{ij}\right)_{m\times m}$}, A satisfies P2 and L (A) = p; p ∈ R for a given matrix A[2]. (2)

Denoted as A ∈ CJ3 (m×m, p).

For example, A = $\begin{pmatrix} 12 & -1 & -5 \\ -2 & 1 & 7 \\ -4 & 6 & 4 \end{pmatrix}$ ∈ CJ3 (3, 6) with L (A) = 6

## 3. Previous Background of Cocentroidal matrices

### 3.1 Centroid of a matrix:

We consider an n × n matrix, $A = [A_1 \quad A_2 \quad \dots \quad \dots \quad A_n]_{1\times n}$

Where each $A_i = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ \vdots \\ a_{ni} \end{bmatrix}_{n\times i}$ for a fixed i =1 to n.

The vector $\overline{OG} = \left(\frac{\sum a_{1i}}{n}, \quad \frac{\sum a_{2i}}{n}, \quad \dots \quad \dots \quad \frac{\sum a_{ni}}{n}\right)$ for all i = 1 to n (3)

is a virtual centroid (G) of the plane containing the points $A_1$, $A_2$ , ….. $A_n$ in $R^n$ [3].

### 3.2 Cocentroidal matrices

The set of all matrices having the same centroid $G^*$ is defined as a set of cocentroidal matrices and is denoted as $G^*(A)$ [4].

## 4. All Cocentroidal matrices associated with the root matrix of Class-1

In this section we try to establish that all cocentroidal matrices associated with the root matrix of Class-1 are the matrices of Class-1. This important derivation conveys the important concepts about similar matrices and eigen values of all such cocentroidal matrices.

Let us consider a matrix, say A of class1. I.e. A ∈ CJ1 (3, L (A))

For Example,

Let $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ ∈ CJ1 (3, 4). i.e. [A is a matrix of Class-1 with Libra value 4.]

Which corresponds to the vectors $\overrightarrow{OP} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, $\overrightarrow{OQ} = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$, and $\overrightarrow{OR} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and let the centroid of its triangular structure in $R^3$ be $G = (\frac{4}{3}, 2, \frac{2}{3})$.

Now, we construct cocentroid matrices (same centroid as that of the root matrix A has) of this matrix A of class1.

We can find infinite triangular structure by keeping the position of the point Q fixed and selecting a variable point R on the line QR.

$R_i = \{R_i = (x_i, y_i, z_i)$ for i = 1, 2, 3, ……$\} \in \overleftrightarrow{QR}$ with the point Q fixed and the point $R_i \neq R$

Then the next part follows taking the point $P_i = P_i (a_i , b_i, c_i)$ in such a way that the centroid of the triangular structure $P_iQR_i$ is the same as that of the structure PQR; i.e. the point G, the one related to root matrix $A^4$. We follow the above mentioned procedure and get some cocentroidal matrices which are listed as follows.

$A_1 = \begin{pmatrix} 12 & -1 & -7 \\ -8 & 4 & 10 \\ 0 & 1 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} 9 & -1 & -4 \\ -5 & 4 & 7 \\ 0 & 1 & 1 \end{pmatrix},$

$A_3 = \begin{pmatrix} 4.5 & -1 & 0.5 \\ -0.5 & 4 & 2.5 \\ 0 & 1 & 1 \end{pmatrix}, A_4 = \begin{pmatrix} 1.5 & -1 & 3.5 \\ 2.5 & 4 & -0.5 \\ 0 & 1 & 1 \end{pmatrix}$ etc… All

This matrices have the same

Centroid $G = (\frac{4}{3}, 2, \frac{2}{3})$.

It is, at this stage, important to note that all this cocentroidal matrices are of Class-1.

We note that in each case $L (A_i) = L (A) = 4$ = Libra value of the root matrix A.

In the same way we can keep the point R fixed and select any point $P_i$, on the line RP. Now, allowing the point G, the centroid, to remain fixed we find the next position of the point Q, say $Q_i$. Following the same routine, we can keep the points P fixed and iterate the procedure.

The important point to be noted and also eye catching is that in all the cases

1. The centroid of the original root matrix does not change
2. Any matrix of this system remains of class1 and that too with the same libra value.

This fact can be easily noted and visualized from the figure given below.

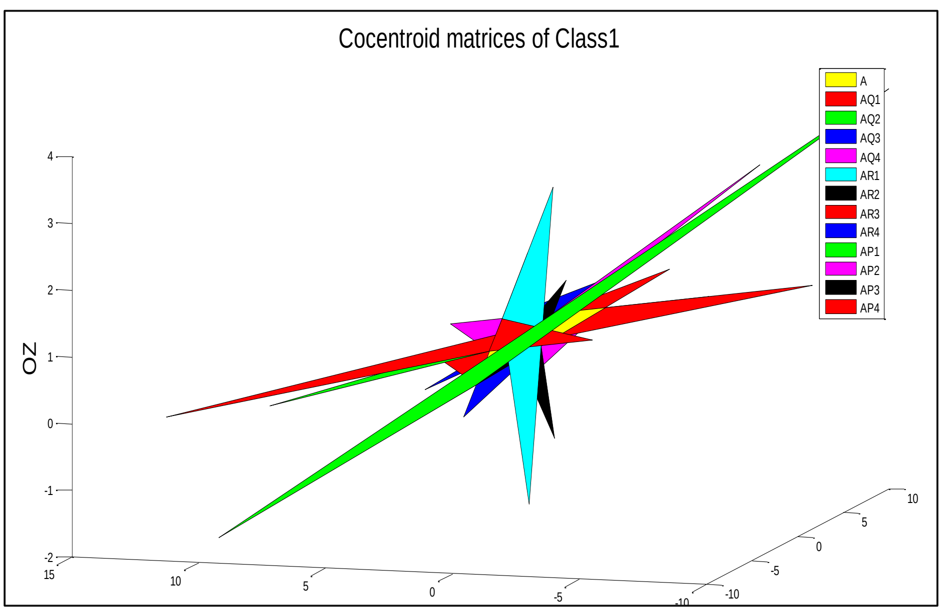


**Fig 1:** Cocentroidal matrices to Class1 matrix

## 5. All Cocentroidal matrices associated with the root matrix of Class-3

In this section we try to establish that all Cocentroidal matrices associated with the root matrix of Class-3 are the matrices of Class-3. This important derivation conveys the important concepts about similar matrices and eigen values of all such cocentroidal matrices.

Let us consider a matrix, say A of class3. i.e. A ∈ CJ3 (3, L (A)

For Example,

Let $A = \begin{pmatrix} 12 & -1 & -5 \\ -2 & 1 & 7 \\ -4 & 6 & 4 \end{pmatrix}$ ∈ CJ3 (3, 6). i.e. [A is a matrix of Class-3 with Libra value 6].

which corresponds to the vectors $\overrightarrow{OP} = \begin{bmatrix} 12 \\ -2 \\ -4 \end{bmatrix}$, $\overrightarrow{OQ} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$, and $\overrightarrow{OR} = \begin{bmatrix} -5 \\ 7 \\ 4 \end{bmatrix}$ and let the centroid of its triangular structure in $R^3$ be G = (2, 2, 2).

Now, we construct cocentroid matrices (same centroid as that of the root matrix A has) of this matrix A of class3.

An infinite set of matrices, (Procedure is discussed in section 4) when Q is fix and K = -2, -1, 0.5, 1.5 etc……..are

$A_1 = \begin{pmatrix} 0 & -1 & 7 \\ 16 & 1 & -11 \\ -10 & 6 & 10 \end{pmatrix}$, $A_2 = \begin{pmatrix} 4 & -1 & 3 \\ 10 & 1 & -5 \\ -8 & 6 & 8 \end{pmatrix}$,

$A_3 = \begin{pmatrix} 10 & -1 & -3 \\ 1 & 1 & 4 \\ -5 & 6 & 5 \end{pmatrix}$, $A_4 = \begin{pmatrix} 14 & -1 & -7 \\ -5 & 1 & 10 \\ -3 & 6 & 3 \end{pmatrix}$ etc…… All this matrices have the same centroid G = (2, 2, 2).

It is important to note that all this cocentroidal matrices are of Class-3.

And in each case L ($A_i$) = L (A) = 6 = Libra value of the root matrix A.

In the same way, When R is fix and P is fix, we get set of infinite cocentroidal matrices of Class3 with the same Libra value.

The important point to be noted and also eye catching is that any matrix of this system remains of class3 and that too with the same libra value.

This fact can be easily noted and visualized from the figure given below.

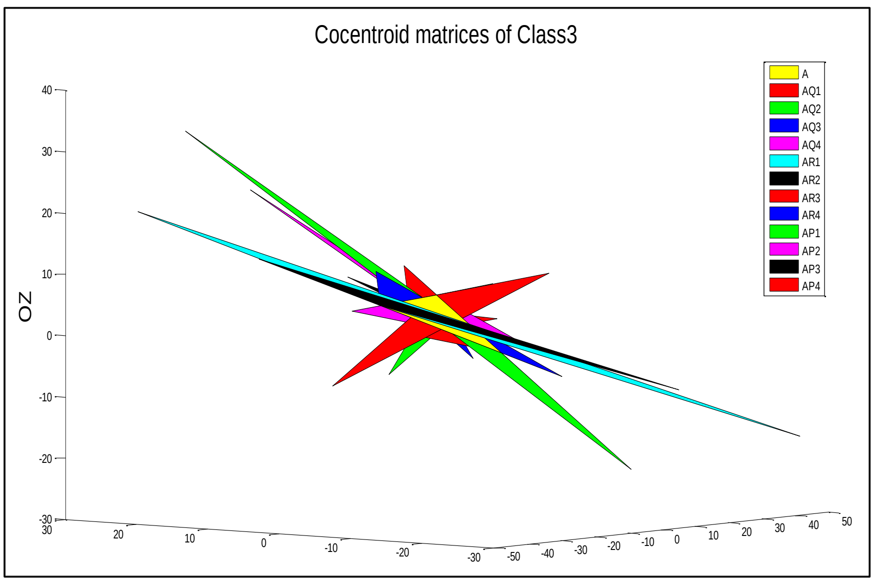


**Fig 2:** Cocentroidal matrices to class3 matrix

## 6. Conclusion

The important criteria that connects the centroid with an infinite class for the two classes under discussion can be extended to different classes which can be justified and some traits of algebraic characteristic continues in the other classes too. In addition to this the problem of meaningful interpretation of eigen values and eigen vectors is an open ended one for the many research minded students to join.

## 7. References


1. Shah SH, Prajapati DP, Achesariya VA, Dr. Jha PJ. Classification of matrices on the Basis of Special Characteristics. International Journal of Mathematics Trends and Technology. 2015; 19(2):91-101.
2. Prajapati DP, Shah SH, Dr. Jha PJ. Commutative Matrices, Eigen Values, and Graphs of Higher Order Matrices preserving Libra Value. International Journal of Applied Research. 2015; 1(12): 246-256.
3. Shah SH, Dr. Jha PJ. Cocentroid Matrices and Extended Metric Space (JS), proceeding of the International conference on Emerging trends in scientific research (ICETSR) C.U.Shah University, Wadhwan, India. 2015:231-234. ISBN: 978-2-642-24819- 9.
4. Shah SH Dr. Jha PJ. Cocentroidal and Isogonal Structures and their Matricinal forms, Procedures and Convergence. International Journal of Mathematics and Statistic Invention. 2016; 4(7):31-39.
5. Shah SH, Achesariya VA, Dr. Jha PJ. Operators on Pythagorean Matrices, IOSR Journal of Mathematics. 2015; 11(3), 51-60.
6. Prajapati DP, Dr. Jha PJ. Norm of a Matrix, Centro-Normal and Centro-Linear Matrices, Proceeding of the International conference on Emerging trends in scientific

research (ICETSR) C.U.Shah, University, Wadhwan, India. 2015; 210-212. ISBN: 978-2- 642-24819-9.
7. Daryl Lynn Stephens. Matrix properties of Magic square. A study, Dissertation. http://faculty.etsu.edu/stephen/matrix-magic-squares.pdf, 1993.
8. Hazra AK. Matrix: Algebra, Calculus and Generalized Inverse, Viva Books Pvt. Ltd, First Indian Edition, 2009. ISBN: 978-81-309-0952-3.
9. Sheth IH. Linear Algebra, Nirav Prakashan. India, 2004.
10. Rana Inder K. An Introduction to Linear Algebra, Ane Books Pvt.Ltd, 2010; 1-15, ISBN: 978-93-8015-696-5
11. Michael Artin. Algebra, Pearson Education, 2007. ISBN: 81-317-1243-5.
12. V Krishnamurthy, VP Mainra, JL Arora. An Introduction to Linear Algebra, East-West Press, Pvt.Ltd, 2006. India. ISBN: 81-85095-15-9