International Journal of Advanced Science and Research
ISSN: 2455-4227
Impact Factor: RJIF 5.12
www.allsciencejournal.com
Volume 2; Issue 5; September 2017; Page No. 98-103

Determination of potential function over orthotropic multiply: Connected infinite medium

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Abstract
The closed form expressions for temperature (Potential) and flux-over multiply connected orthotropic infinite medium by using Fourier transform method are obtained. Flux has Cauchy type singularity at crack tips.

Keywords: fourier transform, integral equations, flux intensity factor

1. Introduction

The fundamental forces of nature are derived from potentials. When mathematical methods are applied we get that the potential function which satisfies the Laplace equation.

How the temperature (potential) behaves on approaching and crossing the fractured boundary? In present research endeavour we shall find the behavior of temperature and flux in steady state conditions. It is assumed that the physical property of medium does not change due to heat conduction. It is also assumed that the temperature & flux vanish as $\sqrt{\alpha^2+\beta^2}\to\infty$ where α and β are two geometrical co-ordinates of a point.

Now a days it is very common that composite materials are replacing the fundamental material from use. The composite materials are treated as orthotropic medium. Though, the orthotropic materials are named due to elastic properties as different in different directions. The materials are made by very thin metallic fibres with matrix. Therefore, the conduction along metallic fibre will be high while perpendicular to fibre will be small.

We know that heat conduction follows the following rule. (a) Heat flows from higher temperature to lower temperature (b) The amount of heat required to produce a given temperature in a body is proportional to the mass of the body and temperature change. The constant of proportionality is called specific heat. In our case the specific heat along α and β axes will be c_1 and c_2 respectively. (c) The rate at which heat flows through an area is proportional to the area and temperature gradient normal to area. The constant of proportionality is called thermal conductivity of material. It is assumed as k_1 and k_2 along α and β axes, respectively.

For steady state conditions and in two dimensions the temperature flow will be satisfying the following partial differential equation

$$\frac{1}{\rho}\left(\frac{k_1}{c_1}\frac{\partial^2}{\partial\alpha^2}+\frac{k_2}{c_2}\frac{\partial^2}{\partial\beta^2}\right)T(\alpha,\beta)=0 \tag{1.1}$$

where ρ is mass density of the medium and T is temperature at general point (α,β).

We make a substitutions in co-ordinate variables as

$$\alpha=\sqrt{\frac{k_1}{c_1}}x,\qquad \beta=\sqrt{\frac{k_2}{c_2}}y \tag{1.2}$$

and then make use of (1.2) in (1.1) it becomes as

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)T(x,y)=0 \tag{1.3}$$

The partial differential equation (1.3) is Laplace equation. Thus we say that the heat conduction in orthotropic medium is called potential function in the form of temperature. The temperature and flux will vanish as $\sqrt{x^2+y^2}\to\infty$ by using (1.2). There are no heat source/sink in the medium.

We are to determine the disturbance due to the presence of cracks in the medium. The cracks occupy the region as $y=0, b<|x|<c, d<|x|<e$ with the prescribed temperature and flux as

$$T(x,0)=\begin{cases} T_1(x), 0\le |x|\le b \\ T_2(x), c\le |x|\le d \\ T_3(x), e\le |x|<\infty \end{cases} \tag{1.4}$$

and

$$\frac{\partial T(x,0)}{\partial y}=\begin{cases} F_1(x), b<|x|<c \\ F_2(x), d<|x|<e \end{cases} \tag{1.5}$$

The cracks occupy the symmetrical places therefore, the conditions (1.4) - (1.5) will reduce to, see figure – 1

$$T(x,0)=\begin{cases} T_1(x), x\in I_1 \\ T_2(x), x\in I_3 \\ T_3(x), x\in I_5 \end{cases} \tag{1.6}$$

$$\frac{\partial T(x,0)}{\partial y}=\begin{cases} F_1(x), x\in I_2 \\ F_2(x), x\in I_4 \end{cases} \tag{1.7}$$

With $I_1=[0,b], I_3=[c,d], I_5=[e,\infty), I_2=(b,c), I_4=(d,e)$

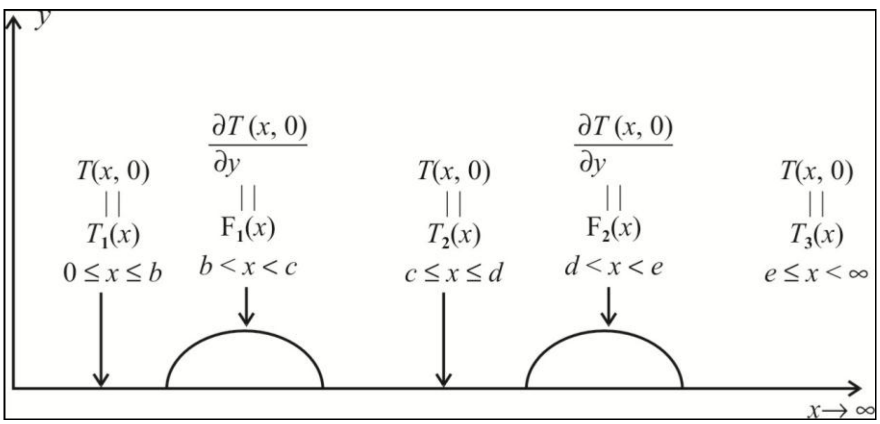

Fig 1: The reduced domain of solution.

There are very few problems over heat distribution. But there is good amount of work done over Thermal-stress. Oleziak [1] had solved for thermal stress caused by penny-shaped crack. Florence and Goddier [2] too, solved for penny shaped crack with linear thermo-elasticity. Shail [3] had solved for thermo-elastic problem in steady state in infinite isotropic solid having external crack. Kassir and Sih [4] solved for thermal stress for external circular crack.

There are few more thesis submitted over thermo-elasticity. Chandra [5], Sorout [6], Singh [7], Singh [8].Singh [9] solved for heat conduction in simply connected domains. The authors Hasanyan *et al.* [10] had discussed cracked plates carrying non-stationary electrical current. They reduced the problem to a system of singular integral equation with Cauchy-type singular Kernels.

Authors Zhong *et al.* [11] discussed that we know that temperature change will affect the overall performance of smart devices, it becomes very important to investigate the responses under thermal loading. Liu [12] discussed the effects of temperature dependent material properties on stress and temperature fields in a cracked metal plate under electric current load.

The plan of the paper is as follows: Section-1 contains the definition and history of potential function along with Temperature as potential functions. The section-2 formulates, reduces and solves the integral equation. Section-3 evaluate the potential function and its derivative in the form of temperature and flux. Section 4 considers a special case of Temperature and Flux. Section-5 concludes as discussion and conclusion. The references are in the last.

2. Formulation Reduction and Solution of Integral Equation

The solution of Laplace equation (1.1) is obtained by using Fourier cosine transform w.r.t. '*x*' as

$$R_c(\xi) = \int_0^\infty R(x)\cos(\xi x)dx$$

with usual inversion. We assume the solution as

$$T(x,y) = \int_0^\infty A(\xi)e^{-\xi y}\cos(\xi x)d\xi \tag{2.1}$$

Then,

$$\frac{\partial T(x,y)}{\partial y} = -\int_0^\infty \xi A(\xi)e^{-\xi y}\cos(\xi x)d\xi \tag{2.2}$$

Thus the boundary conditions (1.2) and (1.3) reduce to

$$\int_0^\infty A(\xi)\cos(\xi x)d\xi = \begin{cases} T_1(x), x \in I_1 \\ T_2(x) \in I_3 \\ T_3(x), x \in I_5 \end{cases} \tag{2.3}$$

and

$$\int_0^\infty \xi A(\xi)\cos(\xi x)d\xi = -\begin{cases} F_1(x),\ x \in I_2 \\ F_2(x),\ x \in I_4 \end{cases} \tag{2.4}$$

The equations (2.3) – (2.4) are quintuple-integral equation. The unknown $A(\xi)$ will be determined by solving the above mixed-boundary value problem.

Solution

We assume the solution, see Kushwaha [13]

$$A(\xi) = \frac{2}{\pi\xi}\left[\left\langle \int_b^c g_1(t) + \int_d^e g_2(t) - \int_0^b T_1'(t) - \int_c^d T_2'(t) - \int_e^\infty T_3'(t)\right\rangle \sin(\xi t)dt\right] \tag{2.5}$$

When (2.5) is substituted in (2.3) and using the integral

$$\int_0^\infty \frac{\sin(\xi t)\cos(\xi x)}{\xi}d\xi = \begin{cases} \pi/2, t > x \\ \pi/4, t = x \\ 0, t < x \end{cases}$$

then it satisfies it if

$$\int_b^c g_1(t)dt = T_1(b) - T_2(c) - T_3(e) \tag{2.6}$$

$$\int_d^e g_2(t)dt = T_2(d) - T_3(e) \tag{2.7}$$

Substituting (2.5) in (2.4) and then using the method of Kushwaha [10] or alternately we can use Kushwaha and Awasthi [14], we get

$$g_1(t) = \frac{2}{\pi^2}\frac{\Delta(t)}{\theta(t)}, t \in I_2 \tag{2.8}$$

$$g_2(t) = -\frac{2}{\pi^2}\frac{\Delta(t)}{\theta(t)}, t \in I_4 \tag{2.9}$$

$$\left.\begin{aligned} &\theta(t) = \left\{\left|t^2 - b^2\right|\left|c^2 - t^2\right|\left|d^2 - t^2\right|\left|e^2 - t^2\right|\right\}^{1/2} \\ &\Delta(t) = \Delta_0(t) + t^2 R + M \\ &\Delta_0(t) = \left\langle \int_b^c F_1(y) - \int_d^e F_2(y) \right\rangle \frac{y\theta(y)}{y^2 - t^2} dy + \left\langle \int_0^b T_1'(\alpha) + \int_c^d T_2'(\alpha) + \int_e^\infty T_3'(\alpha) \right\rangle \frac{\alpha d\alpha}{\alpha^2 - t^2} \end{aligned}\right\} \tag{2.10}$$

where R and M are two arbitrary constants to be determined through (2.6) – (2.7).

3. Physical Quantities

Temperature (Potential)

The temperature $T(x,0)$ is obtained through the value of integral in left hand side of (2.3) and is given as

$$T(x,0) = \begin{cases} \int_x^c g_1(t)dt - T_1(b) + T_2(c) + T_1(0) + T_3(e) \\ \int_x^e g_2(t)dt + T_3(e) \end{cases} \tag{3.1}$$

FLUX (Derivative of Potential)

The flux $\dfrac{\partial T(x,0)}{\partial y}$ is obtained through the value of integral in left hand side of (2.4) and using (2.5) there and then evaluating the integrals, we get.

$$\frac{\partial T(x,0)}{\partial y} = \begin{cases} \dfrac{\Delta(x)}{\pi\theta(x)}, x \in I_1 \\ -\dfrac{\Delta(x)}{\pi\theta(x)}, x \in I_3 \\ \dfrac{\Delta(x)}{\pi\theta(x)}, x \in I_5 \end{cases} \tag{3.2}$$

Where $\theta(x)$ and $\Delta(x)$ are defined in (2.10).We see that in (3.2) flux has Cauchy type or square root singularity at crack tips. We define flux-intensity factor as below (these are defined for stresses as stress-intensity factor at crack tips).

$$T_b' = \lim_{x \to b^-} \sqrt{b - x}\,\frac{\partial T(x,0)}{\partial y} \tag{3.3}$$

$$T_c' = \lim_{x \to c^+} \sqrt{x - c}\,\frac{\partial T(x,0)}{\partial y} \tag{3.4}$$

$$T_d' = \lim_{x \to d^-} \sqrt{d - x}\,\frac{\partial T(x,0)}{\partial y} \tag{3.5}$$

$$T_e' = \lim_{x \to e^+} \sqrt{x-e}\,\frac{\partial T(x,0)}{\partial y} \tag{3.6}$$

Thus using (3.2) in (3.3) – (3.5) we get,

$$T_b' = \frac{\Delta(b)}{\pi\ n_1(b)} \tag{3.7}$$

$$T_c' = -\frac{\Delta(c)}{\pi\ n_1(c)} \tag{3.8}$$

$$T_d' = -\frac{\Delta(d)}{\pi\ n_2(d)} \tag{3.9}$$

$$T_e' = \frac{\Delta(e)}{\pi\ n_2(e)}, \quad T' = \frac{\partial T}{\partial y} \tag{3.10}$$

Where

$$n_1(x) = \sqrt{2x(c^2-b^2)(d^2-x^2)(e^2-x^2)} \tag{3.11}$$

$$n_2(x) = \sqrt{2x(x^2-c^2)(x^2-b^2)(e^2-d^2)} \tag{3.12}$$

4. Special Case

We take one special case. Let

$$T_1(x) = t_1 = \text{constant},\ 0 \le x \le b,\quad T_2(x) = t_2 = \text{constant},\ c \le x \le d,$$
$$T_3(x) = t_3 = \text{constant},\ e \le x < \infty \tag{4.1}$$

$$F_1(x) = F_2(x) = p_0 = \text{constant flux}, \tag{4.2}$$

(4.1) gives that,

$$\frac{dT_1(x)}{dx} = 0,\ \frac{dT_2(x)}{dx} = 0,\ \frac{dT_3(x)}{dx} = 0 \tag{4.3}$$

Using (4.1) - (4.3) into third of (2.10) wet get

$$\Delta_0(t) = p_0\left[\left(\int_b^c - \int_d^e\right)\frac{y\theta(y)dy}{y^2-t^2}\right] \tag{4.4}$$

Now we use (2.6) – (2.7) and (4.1) – (4.4) and solve we get R and M as

$$R = \frac{p_0 S_7 + S_8}{S_9},\ \ M = \frac{p_0 S_{10} + S_{11}}{S_9} \tag{4.5}$$

$$\left.\begin{array}{ll} S_7 = S_3S_4 - S_1S_6, & S_8 = t_4S_6 - t_5S_3 \\ S_9 = S_2S_6 - S_3S_5, & S_{10} = S_2S_4 - S_1S_5 \\ S_{11} = t_3S_5 - t_2S_2, & t_5 = t_2 + t_3, t_4 = t_1 - t_2 - t_3 \end{array}\right\} \tag{4.6}$$

$$S_1 = \left(\int_b^c - \int_d^e\right) y\theta(y) \int_b^c \frac{dt\ dy}{\theta(t)(y^2 - t^2)}, \quad S_2 = \int_b^c \frac{t^2 dt}{\theta(t)}, \qquad S_3 = \int_b^c \frac{dt}{\theta(t)} \tag{4.7}$$

$$S_4 = \left(\int_b^c - \int_d^e\right) y\theta(y) \int_d^e \frac{dt\ dy}{\theta(t)(y^2 - t^2)}, \quad S_5 = \int_d^e \frac{t^2 dt}{\theta(t)}, \qquad S_6 = \int_d^e \frac{dt}{\theta(t)} \tag{4.8}$$

Thus the closed form expressions for temperature distribution at $y = 0$ and flux, too, are evaluated.

5. Conclusion and Discussion

Thus we determined the potential function (Temperature or heat) over multiply connected orthotropic body by using the integral equation method. The method used for heat distribution can be extended to the analysis for crack opening due to heat. The distribution of flux across x-axis for region $x \in I_1 \cup I_3 \cup I_5$ is determined and it is found that flux has square root singularity at crack tips. The singularity at crack tips, it seems, may generate plastic region around crack tips. This type of problems will be discussed in future. The temperature distribution along x-axis for $x \in I_2 \cup I_4$ is smooth; i.e., there is no singularity anywhere in the region. For actual values of Potential and its derivative will be evaluated in terms of variables α and β through (1.2).

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