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Profit analysis of a complex system having two type of failure and rest period of repairman with gamma repair time distribution

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Abstract
The present paper has been designed with an object to analyze and determine some reliability characteristics of a complex system of two non-identical units which is operative with different type of failure and rest period of repairman. The system consists of two units, one is main unit and other is supporting unit. The main unit passes through two types of failure i.e., partial failure mode and total failure mode whereas supporting unit fails directly. A single repairman is always available with the system for the repair a failed unit. The working efficiency of the repairman may reduce on the repairing of totally fail main unit and he needs rest after working some random period of time. The time to failure of the units and the time to which repairman goes for rest are exponentially distributed whereas the completion of rest time of repairman and repair time of the units are gamma distributed with different parameters. Using regenerative point technique, important measures of the system effectiveness are obtained. The graphical behaviours of Profit function have also been studied.

Keywords: reliability, availability, busy period, expected number of repairs, profit analysis, graphical study of model

Introduction
Uncertainty is one of the important issues in management decisions. Two of the most useful uncertainty measures are: system reliability and system availability. Achieving a high or required level of availability is often an essential requisite. Various researchers [2-4, 6-7] including have analysed complex system models under different sets of assumptions such as two types of repair, allowed down time, abnormal weather conditions, random appearance and disappearance of repairman etc. and obtained various economic measures of system effectiveness using the theory of Markov-renewal process, regenerative point technique and supplementary variable technique. Aggarwal [1] had analysed the two units in cold stand by system by considering the two types of failure and single repair time be exponentially distributed. Haggag [5] had analysed the two dissimilar unit cold standby system with three states and preventive maintenance. The common assumption taken in all these systems are that a single repairman repairs the failed unit continuously till it is repaired. But in practice, it is not possible for a repairman to repair a failed unit continuously for a long time due to his tiredness. Thus, the working efficiency of the repairman may reduce and he needs rest after some random period of time.

Taking this fact into consideration, in the present paper we investigate the system model with two non-identical units is assumed to operate in three different modes: normal, partial failure and total failure. The main unit is operative when partial failure occurs and is given minor repair or partial repair to restore it from partial failure mode. When major or total failure occurs, the repairman repairs the main unit and it becomes as good as new. The supportive unit under goes direct failure mode. So, on repairing of the total failure of a main unit, the working efficiency of the repairman may reduce and he needs rest after working some random period of time and he again starts the repair of failed unit which is pre-emptive repeat type i.e., the time already spent in the repair of main unit A goes to waste.

Materials and Methods
The present study gives the probabilistic analysis of a system by making use of regenerative point technique and obtains various measures of system effectiveness such as
1. Transition and steady state probability
2. Mean sojourn time
3. Reliability Analysis and MTSF
4. Availability analysis
5. Expected busy period of the repair
6. Expected number of repair by repairman.
7. Net expected profit earned by the system in (0, t] and in steady state

System Description and Assumptions
1. The system consists of two non-identical units (A and B). Initially, both the units of the system are operative. The system is also functioning with main unit only.
2. The main unit A has three modes- Normal, Partial and Total Failure whereas the supporting unit B has two modes- Normal and Total Failure.
3. The main unit A of the system cannot attain the total failure mode without passing through the partial failure mode. The main unit A is operative when partial failure occurs and is given minor repair or partial repair to restore it from partial failure mode.
4. When the main unit A goes for total failure, then the unit B is idle and the system is in down state. The supportive unit B under goes direct failure mode.

5. It is assumed that after repair of total failure of main unit A, it goes directly to normal mode without passing through the partial failure mode.
6. A single repairman is available to repair both types of failed unit whether the cause is partial or total one. The working efficiency of the repairman may reduce on repairing of total failure of main unit A and he needs rest after working some random period of time and he again starts the repair of failed unit which is pre-emptive repeat type.
7. The main unit A gets the priority in repair over the repair of supportive unit B.
8. The time to failure of the units and the time to which repairman goes for rest are exponentially distributed whereas the completion of rest time of repairman and repair time of the units are gamma distributed with different parameters.

Notation and Symbols for the States of the System

α_1/α_2 : Constant failure rates of unit A from its normal/ partial failure mode.
β : Constant failure rates of unit B
μ/R : Rate of rest of the repairman/ Repairman is in rest period
$\theta_1, \theta_2, \lambda$: Shape parameters of gamma repair time distributions of minor- repair of unit A, major repair of unit A and repair of unit B.
η : Shape parameter of gamma completion time dist. from rest of the repairman.
*/~ : Symbol for Laplace Transform/Laplace-Stieltjes of the function
A_o/B_o : Unit A/B operative and in normal mode.
A_{Pr}/A_{TFr}: Unit Aisin partial/total failure and is under repair.
B_I : Unit B is idle
B_r/B_{wr} : Unit is failed and is under repair /waiting for repair
A_r/A_{wr} : Unit is failed and is under repair /waiting for repair

Transition probabilities and sojourn times:

The steady-state probabilities of transition are given by: $p_{ij} = lim_{t\to\infty} Q_{ij}(t)$. Thus, we have

$$p_{01} = \frac{\beta}{(\alpha_1+\beta)} \quad p_{02} = \frac{\alpha_1}{(\alpha_1+\beta)} \quad p_{10} = \frac{1}{(1+\alpha_1)^{\lambda}} \quad p_{15} = 1 - \frac{1}{(1+\alpha_1)^{\lambda}}$$
$$p_{20} = \frac{1}{(1+\alpha_1)^{\theta_1}} \quad p_{23} = 1 - \frac{1}{(1+\alpha_1)^{\theta_1}} \quad p_{30} = p_{61} = \frac{1}{(1+\mu)^{\theta_2}}$$
$$p_{30} = p_{67} = 1 - \frac{1}{(1+\mu)^{\theta_2}} \quad p_{43} = p_{76} = 1 \quad p_{51} = \frac{1}{(1+\alpha_2)^{\theta_1}}$$
$$p_{56} = 1 - \frac{1}{(1+\alpha_2)^{\theta_1}}$$

It can be easily verified that $p_{01} + p_{02} = 1$ $p_{10} + p_{15} = 1$ $p_{20} + p_{23} = 1$ $p_{30} + p_{34} = 1$ $p_{51} + p_{56} = 1$ $p_{61} + p_{67} = 1$ $p_{43} = p_{76} = 1$

Mean sojourn times

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is: $\Psi_i = E[T_i] = \int P(T_i > t)dt$.

$$\Psi_0 = \int_0^\infty e^{-(\alpha_1+\beta)t}\, dt = \frac{1}{(\alpha_1+\beta)} \quad \Psi_1 = \frac{1}{\alpha_1}\{\frac{1}{(\theta_1+\lambda_2)}\} \quad \Psi_2 = \frac{1}{\alpha_1}\{1 - \frac{1}{(1+\alpha_1)^{\theta_1}}\}$$
$$\Psi_3 = \Psi_6 = \frac{1}{\mu}\{1 - \frac{1}{(1+\mu)^{\theta_2}}\} \quad \Psi_4 = \Psi_7 = \eta \quad \Psi_5 = \frac{1}{\alpha_2}\{1 - \frac{1}{(1+\alpha_2)^{\theta_1}}\}$$

Analysis of Reliability and MTSF

Let the random variable T_i be the time to system failure when system starts up from state $S_i \in E_i$, then the reliability of the system is given by $R_i(t) = P[T_i > t]$. Taking their Laplace transforms and solving the resultant set of equations for $R_0^*(s)$, we get

$$R_0^*(s) = N_1(s)/D_1(s) \tag{1}$$

where, $N_1(s) = (Z_0^* + q_{02}^* Z_2^*)(1 - q_{15}^* q_{51}^*) + q_{01}^*(Z_1^* + q_{15}^* Z_5^*)$ and $D_1(s) = (1 - q_{15}^* q_{51}^*)(1 - q_{02}^* q_{20}^*) - q_{01}^* q_{10}^*$. Taking inverse Laplace Transform of (1), we get reliability of the system. To get MTSF, we use the well-known formula

$$E(T_0) = \int R_0(t) = \lim_{s\to 0} R_0^*(s) = N_1(0)/D_1(0)$$

where,
$N_1(0) = (\Psi_0 + p_{02}\Psi_2)(1 - p_{15}p_{51}) + p_{01}(\Psi_1 + p_{15}\Psi_5)$ and $D_1(0) = (1 - p_{15}p_{51})(1 - p_{02}p_{20}) - p_{01}p_{10}$

Availability Analysis

Define $A_i(t)$ as the probability that the system is up at epoch't' when it initially started from regenerative state S_i. To obtain recurrence relations among different point-wise availabilities $A_i(t)$, we use the simple probabilistic arguments and solving them by taking their Laplace Transform, the L.T. of point–wise availability is given by: $A_0^*(s) = N_1(s)/D_1(s)$. The steady state up time of the units will be given by $A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s A_0^*(s) = N_1(0)/D_1'(0)$. Therefore,

$$N_1(0) = p_{61}p_{30}[p_{10}(\Psi_0 + p_{02}\Psi_2) + p_{01}(\Psi_1 + p_{15}\Psi_5)]$$

and

$$D_1'(0) = p_{61}p_{30}[p_{10}(\Psi_0 + p_{02}\Psi_2) + p_{01}(\Psi_1 + p_{15}\Psi_5)] + p_{02}p_{23}p_{10}p_{61}\Psi_1 + p_{15}\Psi_5(\Psi_3 + p_{34}\Psi_4) + p_{01}p_{15}p_{30}(\Psi_6 p_{56} + p_{53}p_{67}\Psi_7)$$

The expected up time of the system during (0, t] is given by $\mu_{up}(t) = \int_0^t A_0(u)\, du$

Busy Period Analysis

We define $B_i(t)$ as the probability that the repairman is busy in performing the repair of a failed unit when the system initially starts from state $S_i \in E$. In the steady state, the probability that the repairman will be busy in repair is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0}\frac{sN_2(s)}{D_2(s)} = \lim_{s\to 0} N_2(s) \lim_{s\to 0}\frac{s}{D_2(s)} = \frac{N_2(0)}{D_2'(0)}$$

where,
$N_2(0) = p_{01}p_{30}[\Psi_1 p_{61} + p_{15}(\Psi_5 p_{61} + p_{56}\Psi_6)] + p_{02}p_{10}p_{61}(\Psi_2 p_{30} + p_{23}\Psi_3)$
And $D_2'(0) = D_1'(0)$ is same as availability analysis. The expected busy period of the repairman for repair and preventive maintenance during (0, t] is given by $\mu_b(t) = \int_0^t B_0(u)\, du$ so that, $\mu_b^*(s) = B_0^*(s)/s$

Expected Number of Repair by Repairman

Let us define $N_i(t)$ as the expected number of repair of the units by the repairman during the time interval $(0,t]$ when the system initially starts from regenerative state S_i. In the steady state, the probability that the expected number of visits by a repairman for repair is given by $N_0 = \frac{N_3(0)}{D_3'(0)}$ $N_3(0) = p_{61}p_{30}(p_{01} + p_{02}p_{10})$ and $D_3'(0) = D_1'(0)$ is same as in availability analysis.

Profit Function Analysis

Profit function $P(t)$ can easily be obtained for the system model under study with the help of characteristics obtained earlier.

$P(t) =$ Expected total revenue in (0,t] $-$ Expected total expenditure in (0,t] $= C_0\mu_{up}(t) - C_1\mu_b(t) - C_2N_0(t)$

Where, C_0 is revenue per unit up time of the system; C_2 is cost per repair by repairman.

C_1 is cost per unit time for which repairman is busy in repair of the failed unit.

The expected total profits per unit time, in steady state, is $P = C_0A_0 - C_1B_0 - C_2N_0$

Graphical Study of the System Model

For a more clear view of the system characteristics with respect to the various parameters involved, we plot curve for Profit function in figure-2 and figure-3 with respect to the failure parameter (α_1) and repair parameter (θ_1) of unit-A for three different values of repair rate and failure rate respectively, while the other parameters are kept fixed as λ=0.02, α_2=0.06, β=0.04, θ_2=0.36, η=0.40, μ=0.20 C_0=800, C_1=500, C_2=100. From the figure-2, it is observed that Profit decreases as failure rate increases irrespective of other parameters. From the figure-3, it is observed that Profit increases as repair rate increases irrespective of other parameters. The curve also indicates that for the same value of repair rate, Profit is lower for lower values of failure rate.

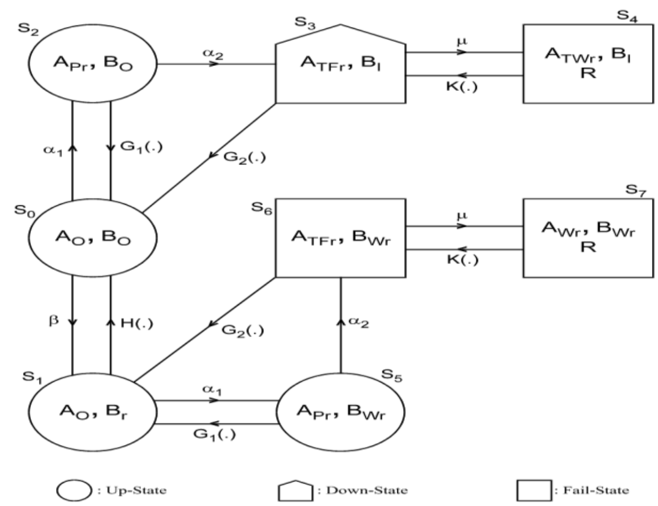

Fig 1: Transition Diagram

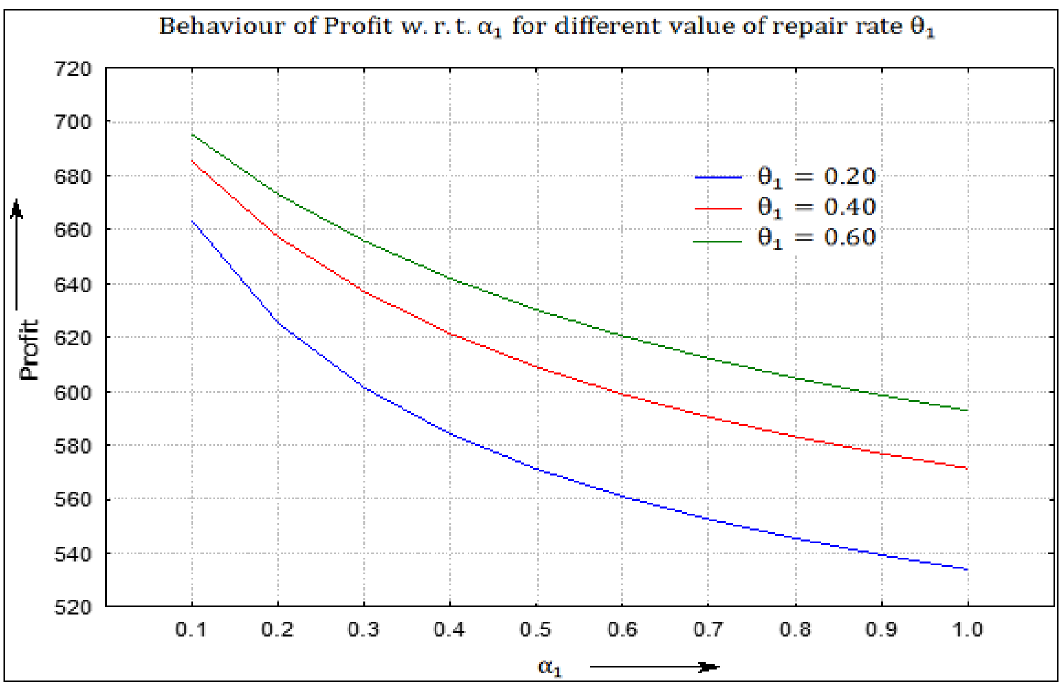

Fig 2

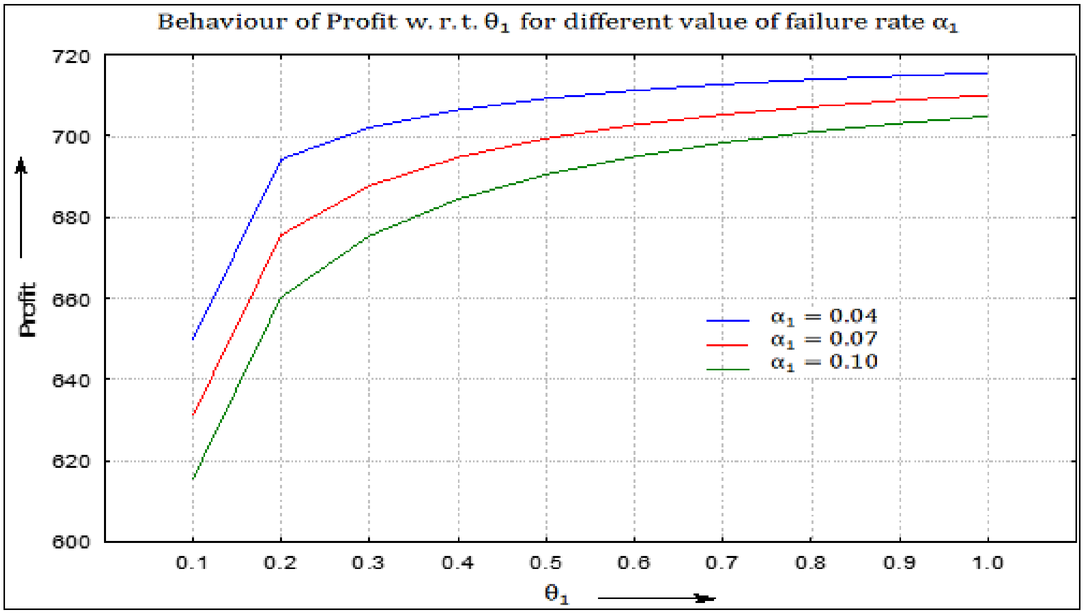

Fig 3

Conclusion

A complex system of two non-identical units which is operative with different type of failure and rest period of repairman has been formulated and analyzed w.r.t. various reliability characteristics. The graphical study of some of the reliability characteristics has also been carried out and hence, it can be concluded that the expected life of the system can be increased by decreasing failure rate and increasing repair rate of the unit which in turn will improve the reliability and hence the effectiveness of the system.

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