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Special Dio 3-tuples for Pronic Number–II

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Abstract
We search for three distinct polynomials with integer coefficients such that the product of any two members of the set subtracted with their sum and increased by a non-zero integer (or polynomial with integer coefficients) is a perfect square.

Keywords: dio 3-tuples, pronic numbers, polynomials

Introduction

Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials n. Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14].

In this communication, we present a few special dio 3-tuples for Pronic numbers of different ranks with their corresponding properties.

Basic definition

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be a special dio 3-tuple with property $D(n)$ if $a_i * a_j - (a_i + a_j) + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non-zero integer or polynomial with integer coefficients.

Method of Analysis

Case 1

Construction of Dio 3-tuples for Pronic number of rank $n-1$ and n.

Let $a = \text{Pro}_{n-1}$, $b = \text{Pro}_n$ be Pronic number of rank $n-1$ and n respectively such that $ab - (a+b) + n^2 + 1$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac - (a+c) + n^2 + 1 = \beta^2 \quad (1)$$

$$bc - (b+c) + n^2 + 1 = \gamma^2 \quad (2)$$

On solving equations (1) and (2), we get

$$(a-b) + (n^2+1)(b-a) = (b-1)\beta^2 - (a-1)\gamma^2 \quad (3)$$

Assume $\beta = x + (a-1)y$ and $\gamma = x + (b-1)y$ and it reduces to

$$x^2 = (a-1)(b-1)y^2 + n^2 \quad (4)$$

The initial solution of the equation (4) is given by

$$x_0 = n^2 - 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - n - 2$

On substituting the values of a and β in equation (1), we get

$$c = 4n^2 - 3 = \text{Pro}_{2n-2} + 6n - 5$$

Hence, The triple $(\text{Pro}_{n-1}, \text{Pro}_n, \text{Pro}_{2n-2} + 6n - 5)$ is a Dio 3-tuple with property $D(n^2+1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 1

n	(a,b,c)	$D(n)$
1	(0,2,1)	2
2	(2,6,13)	5
3	(6,12,33)	10
4	(12,20,61)	17
5	(20,30,97)	26

We present below, some of the Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 2

a	b	c	$D(n)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n - 7$	$D(-n^2+4)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n - 9$	$D(-3n^2+9)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n - 11$	$D(-5n^2+16)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n - 13$	$D(-7n^2+25)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n - 15$	$D(-9n^2+36)$

In general, it is noted that the triple $(\text{Pro}_{n-1}, \text{Pro}_n, \text{Pro}_{2n-2} + 6n - (2k+1))$ is a Dio 3-tuple with the property $D(-tn^2 + t^2)$, where $k = 2, 3, 4, \ldots$ and $t = 1, 2, \ldots$

Case 2

Construction of Dio 3-tuples for Pronic number of rank $n-2$ and n.

Let $a = \text{Pro}_{n-2}$, $b = \text{Pro}_n$ be Pronic number of rank $n-2$ and n respectively such that $ab - (a+b) + 8n^2 - 8n + 6$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac - (a+c) + 8n^2 - 8n + 6 = \beta^2 \quad (5)$$

$$bc - (b+c) + 8n^2 - 8n + 6 = \gamma^2 \quad (6)$$

On solving equations (5) and (6), we get

$$(a-b) + (8n^2 - 8n + 6)(b-a) = (b-1)\beta^2 - (a-1)\gamma^2 \quad (7)$$

Assume $\beta = x + (a-1)y$ and $\gamma = x + (b-1)y$ and it reduces to

$$x^2 = (a-1)(b-1)y^2 + 8n^2 - 8n + 5 \quad (8)$$

The initial solution of the equation (8) is given by

$$x_0 = n^2 - n + 2, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - 4n + 3$

On substituting the values of α and β in equation (5), we get

$$c = 4n^2 - 4n + 5 = \mathrm{Pro}_{2n-2} + 2n + 3$$

Hence, The triple $(\mathrm{Pro}_{n-2}, \mathrm{Pro}_n, \mathrm{Pro}_{2n-2} + 2n + 3)$ is a Dio 3-tuple with property $D(8n^2 - 8n + 6)$.
A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 3

n	(a,b,c)	$D(n)$
1	(0,2,5)	6
2	(0,6,13)	22
3	(2,12,29)	54
4	(6,20,53)	102
5	(12,30,85)	166

We present below, some of the Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 4

a	b	c	$D(n)$
Pro_{n-2}	Pro_n	$\mathrm{Pro}_{2n-2} + 2n + 5$	$D(10n^2 - 10n + 11)$
Pro_{n-2}	Pro_n	$\mathrm{Pro}_{2n-2} + 2n + 7$	$D(12n^2 - 12n + 18)$
Pro_{n-2}	Pro_n	$\mathrm{Pro}_{2n-2} + 2n + 9$	$D(14n^2 - 14n + 27)$
Pro_{n-2}	Pro_n	$\mathrm{Pro}_{2n-2} + 2n + 11$	$D(16n^2 - 16n + 38)$
Pro_{n-2}	Pro_n	$\mathrm{Pro}_{2n-2} + 2n + 13$	$D(18n^2 - 18n + 51)$

Case 3

Construction of Dio 3-tuples for Pronic number of rank $n-2$ and $n-1$.
Let $a = \mathrm{Pro}_{n-2}$, $b = \mathrm{Pro}_{n-1}$ be Pronic number of rank $n-2$ and $n-1$ respectively such that $ab - (a+b) + 3n^2 - 6n + 3$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac - (a+c) + 3n^2 - 6n + 3 = \beta^2 \quad (9)$$
$$bc - (b+c) + 3n^2 - 6n + 3 = \gamma^2 \quad (10)$$

On solving equations (9) and (10), we get

$$(a-b) + (3n^2 - 6n + 3)(b-a) = (b-1)\beta^2 - (a-1)\gamma^2 \quad (11)$$

Assume $\beta = x + (a-1)y$ and $\gamma = x + (b-1)y$ and it reduces to

$$x^2 = (a-1)(b-1)y^2 + 3n^2 - 6n + 2 \quad (12)$$

The initial solution of the equation (12) is given by

$$x_0 = n^2 - 2n + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - 5n + 2$

On substituting the values of a and β in equation (9), we get

$$c = 4n^2 - 8n + 3 = \mathrm{Pro}_{2n-2} - 2n + 1$$

Hence, The triple $\left(\mathrm{Pro}_{n-2}, \mathrm{Pro}_{n-1}, \mathrm{Pro}_{2n-2} - 2n + 3\right)$ is a Dio 3-tuple with property $D(3n^2 - 6n + 3)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 5

n	(a,b,c)	$D(n)$
1	(0,0,-1)	0
2	(0,2,3)	3
3	(2,6,15)	12
4	(6,12,35)	27
5	(12,20,63)	48

We present below, some of the Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 6

a	b	c	$D(n)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2} - 2n + 3$	$D(5n^2 - 10n + 6)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2} - 2n + 5$	$D(7n^2 - 14n + 11)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2} - 2n + 7$	$D(9n^2 - 18n + 18)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2} - 2n + 9$	$D(11n^2 - 22n + 27)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2} - 2n + 11$	$D(13n^2 - 26n + 38)$

Conclusion

In this paper we have presented a few examples of constructing a special Dio 3-tuples for Pronic number of different ranks with suitable properties. To conclude one may search for Dio 3-tuples for higher order Pronic number with their corresponding suitable properties.

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