



Modification in the Polarizability and its effect on ice glaciation

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Abstract

We have estimated nucleation rate of ice particle in external electric field at different temperature by using modified value of polarizability. the effective polarizability increases the Gibb's free energy, nucleation rate and radius of nucleus of ice.

Keywords: surface free energy, nucleation rate, relaxation time

1. Introduction

Ice nucleation is a fundamental cloud process. The process involved in ice formation in cerrus clouds is largely understood only in case of homogeneous freezing of super cooled droplets on water vapour molecules, the polarizability plays a dominant role in inducing the electric dipole. The contribution to polarization of water vapour molecules coming from the electric field generated by droplet dipole have been taken into account. Infact, this contribution would increase the interaction between droplet and water vapour molecules. This would enhance the effect described by murino (1979) ^[1], but the interaction due to this additional factor would not be steady and hence would produce only transient effect.

Singh *et al.* (1986) ^[2] have shown that the resultant electric field on droplet due to external electric field and field induced due to the central dipole, the role of nucleation in water vapour condensation and ice glaciation is about hundred times more near breakdown of dry air as compared to that in absence of electric field. We will evaluate the effect of modified polarizability on glaciation of ice.

Polarizability plays a dominant role in inducing the electric dipole moment of water vapour. A polar molecule is affected by the external electric field in two ways. Firstly, it displaces the centre of gravity of protons and electrons so that an extra dipole moment is induced. This is called the electronic polarizability. Secondly, the dipole tends to orient itself, so that the potential energy is minimum. This process is called orientational polarization. the polarizability depends upon the temperature also. The value of polarizability α is modified ^[3].

$$\alpha_{eff} = \alpha + \frac{p_0^2}{3kT} \quad \dots\dots(1)$$

Where p, k, T are electric dipole moment, Boltzmann constant and temperature respectively. In an external field the moment induced on a droplet is given by

$$\vec{M} = \vec{E} r_i^3 \quad \dots\dots(2)$$

Where E is the induced electric field and r_i is the radius of ice embryo. The moment induced on the molecule in presence of cloud is given by

$$\vec{M} = \alpha \vec{E} \quad \dots\dots(3)$$

Where α and k is polarizability and Boltzmann constant respectively.

The contribution due to permanent electric dipole moment to the polarizability is $\frac{p_0^2}{3kT}$.

2. Theoretical Methodology

2.1 Without Electric Field

The Gibb's free energy for the formation of ice embryo is given by

$$\Delta G_i = -\frac{4}{3} \pi r_i^3 \Delta G_v + 4 \pi r_i^2 \sigma_{i/v}$$

Where
$$\Delta G_v = \frac{\rho_i R T \ln S_{vi}}{M_w}$$

ρ_i = Density of ice

R = Universal gas constant

S_{vi} = Supersaturation ratio of water vapour over ice surface

$\sigma_{i/v}$ = Surface free energy of ice vapour interface

M_w = Molecular weight of water

The surface free energy of ice vapour interface $\sigma_{i/v}$ has been calculated by Antonow`s equation (Smitha *et al.* 1986) [4].

$$\sigma_{i/v} = \sigma_{i/w} + \sigma_{w/v} \tag{4}$$

Here $\sigma_{i/w} = 33 \text{ dynes/cm}$ and it is nearly independent of temperature but $\sigma_{w/v}$ is dependent on temperature. The value of $\sigma_{w/v}$ and $\sigma_{i/v}$ are used as in table 1 (Edward *et al.* 2003) [5].

The critical radius is obtained by setting

$$\frac{\partial}{\partial r_i} (\Delta G_i) = 0 \tag{5}$$

The radius of critical ice nucleus is

$$r_i^* = \frac{2 M_i \sigma_{i/v}}{\rho_i R T \ln S_{vi}} \tag{6}$$

The energy of formation of a critical nucleus is

$$\Delta G_i^* = \frac{4}{3} \pi \sigma_{i/v} r_i^{*2} \tag{7}$$

The number of ice molecules in critical nucleus is given by

$$n_i^* = \frac{\frac{4}{3} \pi r_i^{*2} \rho_i N}{M_i} \tag{8}$$

2.2 In Presence of External Electric Field

Water is strongly polarizable having dipole moment 1.81×10^{-18} esu. In an external electric field the water embryo rapidly increases in size. The expression for the rate of growth of radius of ice embryo has been derived as (Singh *et al.* 1986).

$$\frac{dr_i}{dt} = \frac{\rho_v}{\rho_i} \left(\frac{9 \alpha_{eff} \lambda E^2}{m_w r_i} \right)^{1/2}$$

Where

m_w = Mass of water vapour molecule

λ = Mean free path

E = External electric field

ρ_v = Density of water vapour

Integrating above equation with in the limit $r_i = 0$ to r_i^* (critical radius of ice nucleus in presence of electric field) and $t = 0$ to $t = \tau$ (relaxation time)

We get

$$r_i^* = \left[\frac{3 \rho_v \left(\frac{\alpha_{eff} \lambda E^2}{m_w} \right)^{1/2} \tau}{2 \rho_i} \right]^{2/3} \dots(9)$$

This is the modified value of radius of critical nucleus in ice phase, considering the effect of electric field and using modified value of polarizability.

Accordingly, Gibb’s free energy ΔG_i^* and number of ice molecules in critical nuclei are also modified. Pruppacher and Klett (1978) [6] derived the expression for nucleation rate of water. The nucleation rate for ice phase is given by similar expression and we get

$$\ln J_i^* \propto \frac{\Delta G_i}{kT} \dots\dots\dots(10)$$

In the present calculation we have taken

- R = 8.317 x 10⁷ erg deg K⁻¹ mole⁻¹
- N = 6.025 x10²³ / mole
- k = 1.38 x10⁻¹⁶ erg deg K⁻¹
- ρ_i = 0.917 gm cm⁻³
- M_i = 18
- m_w = 3.0 x 10⁻²³ gm
- λ = 10⁻⁵ gm cm⁻³
- p_0 = 1.81 x 10⁻¹⁸ esu
- ρ_v = 10⁻⁵ gm cm⁻³
- α = 5 x 10⁻²³ cm³

$\sigma_{i/v}$ is used as in table 1.

Calculated values of r_i^* , n_i^* , ΔG_i^* and $\ln J_i^*$ at E = 10 esu as a function of relaxation time τ at temperature 273K, 283K and 293K are reported in table 2.

At a constant electric field, the values of radius, number of molecules, Gibb’s free energy and nucleation rate of a critical nucleus for ice phase are formed to increase with increase in relaxation time.

Thus polarizability of water vapour molecules plays an important role in the ice glaciation in presence of external electric field.

3. Table

Table 1: Variation of $\sigma_{w/v}$ and $\sigma_{i/v}$ with temperature

Temperature T(K)	Surface free energy of water vapour interface $\sigma_{w/v}$ (dyne/cm)	Surface free energy of ice vapour interface $\sigma_{i/v}$ (dyne/cm)
273	75.7	108.7
283	74.2	107.2
293	72.8	105.8

Table 2: Calculated values of radius (r_i^*), number of ice molecules (n_i^*), gibb’s free energy (ΔG_i^*) and nucleation rate ($\ln J_i^*$) at electric field E = 10 esu. As a function of relaxation time (τ) in ice phase

Temperature T(K)	τ μ sec	$r_i^* \times 10^{-8}$ cm	n_i^*	ΔG_i^*	$\ln J_i^*$
273	8	7.40	52.09	2.49	66.20
	16	11.75	208.37	6.28	166.80
	24	15.40	468.83	10.79	286.42
	32	18.67	833.47	15.83	420.32
	40	21.65	1302.30	21.32	569.97

283	8	7.37	51.42	2.44	62.43
	16	11.70	205.67	6.14	157.32
	24	15.33	462.75	10.55	270.12
	32	18.57	822.66	15.48	396.41
	40	21.55	1285.41	20.85	533.77
293	8	7.34	50.79	2.39	59.02
	16	11.65	203.15	6.01	148.73
	24	15.27	457.08	10.33	255.39
	32	18.50	812.59	15.15	374.79
	40	21.45	1269.67	20.40	504.66

4. Result and discussion

The computed results for radius (r_i^*), number of ice molecules (n_i^*), gibb's free energy (ΔG_i^*) and nucleation rate ($\ln J_i^*$) at electric field $E = 10$ esu are given in table 2 at temperature 273, 283 and 293 as a function of relaxation time (τ). It is clear that with increasing the relaxation time (τ) the radius (r_i^*), number of ice molecules (n_i^*), gibb's free energy (ΔG_i^*) and nucleation rate ($\ln J_i^*$) increases very rapidly.

5. References

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