



## Part A: Characteristics and special role of matrices of Class 4B

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### Abstract

We have classified square matrices based on the 'Libra value' \* property in six different classes. In this note we unfold what we have found and establish about dominating properties of matrices of class 4B\*\* and also its ring structure. It has been shown that some sets of standard form of class 4B matrices satisfy commutative property under matrix multiplication. In addition to this, we describe different transformations that convert matrices of class 4B to their corresponding matrices of class 4A and vice – versa.

**Libra value:** Libra value of a given square matrix is that constant value for the given matrix obtained by adding the all the entries of each row or column or diagonal (both leading or non-leading) as the case may be depending on the classification.

**Notations:** (1) CJ4A ( $n \times n$ ,  $L(A) = \_$ ) [Square Matrix of class 4A with order  $n \times n$  and  $L(A) =$  libra value.  $L(A) =$  libra value]

(2) CJ4B ( $n \times n$ ,  $L(A) = \_$ ) [Square matrix of class 4B with order  $n \times n$ ] and  $L(B) =$  libra Value]

(3) CJS4B [A symmetric matrix of CJ4B]

(4)  $\bar{0}$  is a Null matrix

(5) I is an identity matrix

**Keywords:** class – 4B, class 4A, libra value, eigen values

### 1. Introduction

Introduction to classification and general form: In this introductory phase we describe the classification of square matrices according to the properties – characteristic properties, according to libra value – p which is a real or complex constant. We give their general form of each class.

#### 1.1 CJ1: Class – J1

Square matrices falling in this class possess the property that the sum of all the entries of each column remains the (same) constant; we name this property as  $P_1$ .

All square matrices possessing  $P_1$  fall under the class 1, denoted as CJ1.

Without loss of generality, for any given constant P, we write the general form of this

$$CJ1 = \{A | A \in M_{n \times n}, \text{ with } L(A) = p\}$$

Where  $L(A) = p$  is the libra value of the matrix A.

e.g.

$$A = \begin{bmatrix} a & c & e \\ b & d & f \\ p-(a+b) & p-(c+d) & p-(e+f) \end{bmatrix} \quad (1)$$

Where the letters 'a' through 'f' are any real values and 'p' is libra value.

If 'p' is pre- known then this matrix has six degree of freedom.

$$A \in CJ1(3 \times 3, L(A) = p)$$

For a given value p (say  $p = 9$ ), we give an illustration.

For free choice of real values through ‘a’ up to ‘f’

$$A = \begin{pmatrix} 2 & -3 & 3 \\ 4 & 5 & 7 \\ 3 & 7 & -1 \end{pmatrix} \text{ which belongs to class 1.}$$

**1.2 CJ2: Class – J2**

Square matrices falling in this class possess the property that the sum of all the entries of each row remains the (same) constant; we name this property as P<sub>2</sub>.

All square matrices possessing P<sub>2</sub>, fall under the class 2, denoted as CJ2.

Without loss of generality, for any given constant p, we write the general form of this class.

CJ2 = { B | B ∈ M<sub>n×n</sub> with L(B) = p }. Where L(B) = p is the libra value of the matrix B.

e.g.

$$B = \begin{bmatrix} a & b & p-(a+b) \\ b & d & p-(c+d) \\ e & f & p-(e+f) \end{bmatrix} \dots (2)$$

Where the letters ‘a’ through ‘f’ are any real values and ‘p’ is libra value. If ‘p’ is pre-known then this matrix has six degree of freedom.

$$B \in CJ2(3 \times 3, L(B) = p)$$

For a given value p (say p = 9), we give an illustration.

For free choice of real values through ‘a’ up to ‘f’

$$B = \begin{bmatrix} 2 & -3 & 10 \\ -\frac{3}{2} & \frac{7}{2} & 7 \\ -1 & 4 & 6 \end{bmatrix} A \in CJ2(3 \times 3, L(B) = 9)$$

**1.3 CJ3: Class – J3**

Square matrices falling in this class possess the property that the sum of all the entries of each column and each row elements remains the (same) constant; we name this property as P<sub>3</sub>.

All square matrices possessing P<sub>3</sub>, fall under the class 3, denoted as CJ3.

Without loss of generality, for any given constant p, we write the general form of this class as

CJ3 = { C | C ∈ M<sub>n×n</sub> with L(C) = p }. Where L(C) = p is the libra value of the matrix C.

e.g.

$$C = \begin{bmatrix} a & b & p-(a+b) \\ c & p-(b+c) & b \\ p-(a+c) & c & a \end{bmatrix} \text{ OR } C = \begin{bmatrix} a & b & p-(a+b) \\ c & p-(b+c) & a \\ p-(a+c) & a+c-b & b \end{bmatrix} \dots (3)$$

Where the letters through ‘a’ up to ‘c’ stand for real numbers. Also p ∈ R is a given

$$\text{real number. } C \in CJ3(p_3, L(C) = p)$$

For a given value p (say p = -10), and free choice for ‘a’, ‘b’ and ‘c’

[say a = 5, b = -3, c = 1/2]

$$C = \begin{bmatrix} 5 & -3 & -12 \\ \frac{1}{2} & -8 & -3 \\ \frac{-31}{2} & \frac{1}{2} & 5 \end{bmatrix} \quad C \in CJ3(3 \times 3, L(C) = -10)$$

#### 1.4 Classes CJ4A and CJ4B

The above mentioned classes are each an extension to the above mentioned class CJ3. We further, add two additional condition, one for getting each class 4A and 4B. At this stage it is essential to state the generator properties of the classes 4A and 4B.

##### CJ4A:

We state the properties  $P_{4A}$ ; square matrix for which the sum of all elements of each row, all the elements of each column, and all the elements of non leading (secondary) diagonal remains the (same) constant then the square matrix is said to hold the property  $P_{4A}$ . The set of all square matrices possessing the property  $P_{4A}$  generates the class of square matrices denoted as ‘CJ4A’.

$$CJ4A = \{D | D \in M_{n \times n}, D \text{ satisfies } P_{4A}\}$$

$$D \in CJ4A\{n \times n | L(D) = p\}$$

(4)

The general format of a  $3 \times 3$  matrix of class 4A,(CJ4A), is as follows.

$$D = \begin{bmatrix} p-(a+b) & a & b \\ b^* & p-(a+b) & a \\ a^* & b & p-(a+b) \end{bmatrix}$$

(5)

For any two arbitrary real values ‘a’ and ‘b’ and pre-known (given) real constant p, we write

$$D \in CJ4A(p_{4A}, L(D) = p)$$

$b^*$  and  $a^*$  are real values so that  $b^* + a^* = b + a$ .

We take an illustration.

For given p = 8 say, a = 5, b = 6 and  $a^* = 7, b^* = 4$  ( $\because a + b = a^* + b^*$ )

we have,

$$D = \begin{bmatrix} 8 - (5 + 6) & 5 & 6 \\ 4 & -5 & 9 \\ 7 & 8 & -7 \end{bmatrix} = \begin{bmatrix} -3 & 5 & 6 \\ 4 & -5 & 9 \\ 7 & 8 & -7 \end{bmatrix}$$

$$D \in CJ4A(3 \times 3, L(A) = 8)$$

##### CJ4B:

We have classified square matrices based on the ‘libra value’ \* property in six different classes. In this note we unfold what we have found and established about dominating properties of matrices of class 4B . In addition to this, we describe different transformations that result matrices of class 4B to their corresponding matrices of class 4A and vice – versa.

This main focus of the content of this paper is on this class 4B (CJ4B).

We state the property  $P_{4B}$ ; square matrix for which the sum of all elements of each row, all the elements of each column, and all the elements of leading (principal) diagonal remains the (same) constant then the square matrix is said to hold the property  $P_{4B}$ .

The set of all square matrices possessing the property  $P_{4B}$  generates the class of square matrices denoted as ‘CJ4B’.

$$CJ4B = \{D | D \in M_{n \times n}, D \text{ satisfies } P_{4B}\}$$

$$D \in CJ4B\{P_{4B} | L(D) = p\}$$

(6)

The general format of a  $3 \times 3$  matrix of class 4B (CJ4B), is as follows.

The different forms of the square matrices falling in this class are described below.

1. For a given real p (Libra Value) and with arbitrary choice of real values 'a, b and c' the first form we write as follows.

$$B = \begin{bmatrix} p - (a + b) & b & a \\ 2p - (2a + b + 2c) & -p + (2a + b + c) & c \\ -2p + 3a + 2b + 2c & 2p - (2a + 2b + c) & p - (a + c) \end{bmatrix}$$

(7)

$$B \in CJ4B(3 \times 3, L(A) = p)$$

e.g. For (say)  $p = 2, a = 1, b = -2$  and  $c = 4$

$$B = \begin{bmatrix} 3 & -2 & 1 \\ -4 & 2 & 4 \\ 3 & 2 & -3 \end{bmatrix}$$

We have  $L(B) = 2$

One more form, generalized in nature, is as follows.

$$B_1 = \begin{bmatrix} p + k & p - k & p + k \\ p + k & p & p \\ p - k & p + 2k & p \end{bmatrix}$$

(8)

With  $B_1 \in CJ4B(3 \times 3, L(A) = 3p + k)$

Where  $p$  and  $k \in R$

**Comments**

1. The matrices pursuing this nature of formation give rise to an infinite class. We, in most of the cases, shall follow this form. We can have different forms of above type given by (8)
2. It is a notable fact that of order  $3 \times 3$ , i.e.  $A = (a_{ij}) i, j = 1, 2, 3$ . In our case as shown in (5). In addition to the property  $p_{4B}$ , we have  $a_{12} + a_{13} = a_{21} + a_{31}$

$$B_1 = \begin{bmatrix} 7 & -3 & 7 \\ 7 & 2 & 2 \\ -3 & 12 & 2 \end{bmatrix}$$

$B_1$  has a libra value,  $L(B_1) = 11$

In order to reflect on the comments above; we have one more general form as follows.

$$B_2 = \begin{bmatrix} p + k & p - k & p + k \\ 3p - k & k & k \\ -p + k & 2p + k & 2p - k \end{bmatrix}$$

(9)

With  $p, k \in R$ .

$$B_2 \in CJ4B(3 \times 3, L(A) = 3p + k)$$

Again, we take  $p = 2$  and  $k = 5$ , so we have

$$B_2 = \begin{bmatrix} 7 & -3 & 7 \\ 1 & 5 & 5 \\ 3 & 9 & -1 \end{bmatrix} \in CJ4B(3 \times 3, L(B_2) = 11)$$

Note that  $a_{12} + a_{13} = a_{21} + a_{31}$

### 1.5 Next higher order $4 \times 4$ of class 4B

We show the some forms of  $4 \times 4$  matrices of class 4B. some basic notions are as follows. Let  $p \in R$ , which will remain libra value of the matrix under construction.

In this case of  $4 \times 4$  matrix we have 16 entries with 4 row conditions 4 column conditions and one condition on the entries of leading diagonal. On the top of all these conditions are controlled by the libra value of the matrix. These 16 variables are involved in 9 equations which are not all independent. There are 6 free variables and remaining variables are constrained with libra value. Let the free variables be denoted by the letters through 'a up to f'.

Let  $p_1, p_2$  and  $p_3$  be the real entries so that  $p_1 + p_2 + p_3 = a + b + c$ . While the entries marked by \* notations are controlled by libra value  $L(A) = p$ .

In this case the general form is

$$A = \begin{bmatrix} a & b & c & *p - (a + b + c) \\ * & d & e & p_1^{**} \\ * & f & * & p_2^{**} \\ * & * & * & p_3^{**} \end{bmatrix} \dots\dots (10)$$

[some \* and \*\* entries are also controlled by the libra value.]

e.g. Let  $L(A) = 11 = p$

'A' is the matrix to be constructed that possesses the properties of class 4B.

Let  $a = 2, b = 3, c = 5, d = -3, e = 4, f = -2$ .

Following the above instructions the matrix A of class 4B is as follows.

$$A = \begin{bmatrix} 2 & 3 & 5 & *1 \\ *12 & -3 & 4 & ** -2 \\ * -9 & -2 & *11 & **11 \\ *6 & *13 & * -9 & **1 \end{bmatrix}$$

We have  $C \in CJ4A(4 \times 4, L(C) = p)$

We give an example to show the method.

### Mathematical Insight for the above construction

1) There are 4 row conditions, 4 column conditions and 1 condition on the main diagonal.

Thus we have  $4 + 4 + 1 = 9$  equations.

All this conditions are governed by the libra value of the matrix A of class 4B. we have  $L(A) = p$ .

The first row condition, the last column condition and one leading row conditions are dependent conditions. This makes six line only independent equations. Thus there 10 conditions are controlled by libra value and 6 variables are of free choice; they are a, b, c, d, e, and f in the work shown above.

One of the general forms of class 4A matrix is as follows.

For any two real values p and k

$$A = \begin{bmatrix} p+2k & -p-k & 2p-k & p+k \\ -p-k & 3p+2k & 2p-2k & -p+2k \\ 2p-k & -p+k & 5k & 2p-4k \\ p+k & 2p-k & -p-k & p+2k \end{bmatrix}$$

..... (11)

$$A \in CJ4A(4 \times 4, L(A) = 3p+k)$$

(1) For  $p = 4$  and  $k = \frac{1}{2}$

$$A = \begin{bmatrix} 5 & -\frac{9}{2} & \frac{15}{2} & \frac{9}{2} \\ -\frac{9}{2} & 13 & 7 & -3 \\ \frac{15}{2} & -\frac{7}{2} & \frac{5}{2} & 6 \\ \frac{9}{2} & \frac{15}{2} & -\frac{9}{2} & 5 \end{bmatrix} \text{ with } L(A) = \frac{25}{2}$$

(2) \* For  $p = 3$  and  $k = 1$

$$A = \begin{bmatrix} 5 & -4 & 5 & 4 \\ -4 & 11 & 4 & -1 \\ 5 & -2 & 5 & 2 \\ 4 & 5 & -4 & 5 \end{bmatrix} \text{ with } L(A) = 10$$

As we have seen that matrices of class 4A can be converted to matrices of class 4B by any one of the above three transformations, we apply the following rule, we apply the following rule;  $T(a_{ij}) = a_{5-j,i}$  for  $i$  and  $j = 1,2,3,4$  [‘,’ Sign is introduced to avoid the concatenation of the first two suffices] Applying this, the given matrix ‘A’ of class 4A is converted in to a matrix ‘A<sub>1</sub>’ of class 4B. It is written as follows.

$$A_1 = \begin{bmatrix} p+k & -p+2k & 2p-4k & p+2k \\ 2p-k & 2p-2k & 5k & -p-k \\ -p-k & 3p+2k & -p+k & 2p-k \\ p+2k & -p-k & 2p-k & p+k \end{bmatrix}$$

(12)

$$A_1 \in CJ4B(4 \times 4, L(A_1) = 3p+k)$$

We note one fact that by applying this transformation the libra value is preserved.

i.e.  $L(A) = L(A_1) = 3p+k$  for  $p, k \in R$

E.g. for  $p = 3, k = 1$

$$A_1 = \begin{bmatrix} 4 & -1 & 2 & 5 \\ 5 & 4 & 5 & -4 \\ -4 & 11 & -2 & 5 \\ 5 & -4 & 5 & 4 \end{bmatrix}$$

$$A_1 \in CJ4B(4 \times 4, L(A_1) = 10)$$

Note: One more transformation in the same style will transform the matrix  $A_1$  of class 4B is converted to the corresponding one of class 4A. It preserves the libra value.

$$A_1 = \begin{bmatrix} 5 & -4 & 5 & 4 \\ 2 & 5 & -2 & 5 \\ -1 & 4 & 11 & -4 \\ 4 & 5 & -4 & 5 \end{bmatrix}$$

**2. Characteristics and Algebra on class 4B**

We discuss characteristics of the member matrices of class 4B.

To study the properties inherent in the matrices of class 4B, we shall take the following standard forms of matrices of class 4B. Some of the standard forms of the class 4B that we have been are as follows.

$$(1) A_1 = \begin{bmatrix} a & b & p - (a + b) \\ 2a - b & p - 2a & b \\ p + b - 3a & -p + 2a & a \end{bmatrix}, L(A) = p \dots\dots\dots (13)$$

$$(2) A_2 = \begin{bmatrix} p - (a + b) & b & a \\ 2p - (2a + b + 2c) & -p + (2a + b + c) & c \\ -2p + 3a + 2b + 2c & 2p - (2a + 2b + c) & p - (a + c) \end{bmatrix}_{3 \times 3}, L(A) = p \dots\dots\dots (14)$$

$$(3) A_3 = \begin{bmatrix} p + k & p - k & p + k \\ p + k & p & p \\ p - k & p + 2k & p \end{bmatrix}_{3 \times 3} \dots\dots\dots (15)$$

$A_3 \in CJ4B(3 \times 3, L(A_3) = 3p + k)$   
 With p and k ∈ R

For p = 2, k = 5, we given an illustration of the form given above.

$$(4) A_4 = \begin{bmatrix} 7 & -3 & 7 \\ 7 & 2 & 2 \\ -3 & 12 & 2 \end{bmatrix}_{3 \times 3}$$

$A_4 \in CJ4B(3 \times 3, L(A) = 3(2) + 5 = 11)$

**2.1 Some Observations**

During the ongoing work, we have some points pertaining to class 4B.

(1) The null matrix, denoted as **0** is a member of class 4B.

$\mathbf{0} = (a_{ij} = 0)$  for all  $i, j = 1$  to  $n$   
 [Here we take  $i, j = 1, 2$  and  $3$ ]

$$\mathbf{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in CJ4B(3 \times 3, L(A) = p = 0)$$

(2) The identity matrix, denoted as  $I_n$ , is not a member of class 4B.

$$I_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

For n = 3,

$$I_3 = I_{3 \times 3} \notin CJ4B(3 \times 3, L(A) = 0)$$

## 2.2 Three Fundamentals

In order to work further in the set CJ4B we shall accept the three fundamental tenets which are already accepted in general

### 1) Equality of Two Matrices

$$A = (a_{ij}) \in CJ4B(3 \times 3 (= n \times n), L(A) = 3p_1 + k_1)$$

$$B = (b_{ij}) \in CJ4B(3 \times 3 (= n \times n), L(B) = 3p_2 + k_2)$$

They are said to be equal,  
i.e.  $A = B$

$$\Leftrightarrow \forall \text{ real values } p_1, p_2, k_1 \text{ and } k_2$$

$$p_1 = p_2 \text{ and } k_1 = k_2$$

$$\Rightarrow a_{ij} = b_{ij} \text{ for all } i \text{ and } j.$$

### 2) Additional of Two Matrices

If there are any two matrices of same order of class 4B, then this rule allows you to add the two matrices only if the addition is only of corresponding element. This will result in to a matrix of the class 4B only.

If  $A \in CJ4B(3 \times 3 (= n \times n), L(A) = 3p_1 + k_1) = (a_{ij})$

and  $B \in CJ4B(3 \times 3 (= n \times n), L(B) = 3p_2 + k_2) = (b_{ij})$

then  $A + B = C = (c_{ij} = a_{ij} + b_{ij})$

Also  $C \in CJ4B(3 \times 3, L(C) = L(A) + L(B))$

e.g.  $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 4 & 3 \\ 4 & 3 & -2 \end{bmatrix} \in CJ4B(3 \times 3, L(A) = 5)$

$$B = \begin{bmatrix} 5 & 3 & -1 \\ 3 & -1 & 5 \\ -1 & 5 & 3 \end{bmatrix} \in CJ4B(3 \times 3, L(B) = 7)$$

$$A + B = \begin{bmatrix} 8 & 1 & 3 \\ 1 & 3 & 8 \\ 3 & 8 & 1 \end{bmatrix} \in CJ4B(3 \times 3, L(A + B) = 12)$$

Then

note that  $L(A + B) = L(A) + L(B)$

**3) Multiplication by a scalar**

For any matrices  $A(a_{ij}) \in CJ4B(3 \times 3 (= n \times n), L(A) = 3p_1 + k_1)$

and  $c \in R$ , we define multiplication of A by the scalar c denoted symbolically as cA.

it is matrix such that

$$cA = c(a_{ij}) = (ca_{ij})$$

cA is obtained by multiplying each element of A by the factor c,  $c \in R$ .

$$cA \in CJ4B(3 \times 3 (= n \times n), L(A) = cL(A))$$

$$\text{e.g. } A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 4 & 3 \\ 4 & 3 & -2 \end{bmatrix} \in CJ4B(3 \times 3, L(A) = 5)$$

$$\text{for } c = 2, cA = \begin{bmatrix} 6 & -4 & 8 \\ -4 & 8 & 6 \\ 8 & 6 & -4 \end{bmatrix} \in CJ4B(3 \times 3, L(cA) = 2 \times 5 = 10)$$

the same rules hold true for the matrices of higher order of the class CJ4B.

now, in the next algebra of class 4B.

**2.3 CJ4B – An Abelian group under ‘+’**

**Theorem 1: To show that A subset of class4B whose members are in the standard format is an Abelian structure (group).**

In this section, we consider the format of the matrix A as follows

$$A = \begin{bmatrix} p+k & p-k & p+k \\ p+k & p & p \\ p-k & p+2k & p \end{bmatrix} \dots\dots\dots (16)$$

For all real value of p and k, it forms an infinite set denoted as CJ4B.

$$CJ4B = \{A|A \text{ is order } 3 \times 3, L(A) = 3p + k\} \dots\dots\dots (17)$$

We have already defined operation ‘+’ which follows regular addition process that already exists on matrix system. [As shown in section 2.1.1, we write the conditions forming an abelian group under ‘+’]

**1) Closure Property**

The operation ‘+’ is closed under matrix addition of the member matrices of CJ4B.  
i.e. for  $A, B \in CJ4B$  we have  $A + B \in CJ4B$ .

**2) Associative Property**

The set CJ4B obeys Associative property under ‘+’.  
i.e. for  $A, B$  and  $C \in CJ4B$ .  $A = (a_{ij})$ ,  $B = (b_{ij})$ , and  $C = (c_{ij})$  are of the same order.  
Let  $L(A) = p_1$ ,  $L(B) = p_2$ , and  $L(C) = p_3$ . all real entries;  
Since  $a_{ij}$ ,  $b_{ij}$ , and  $c_{ij}$  are real entries and associative law holds in the set of real values in R. we have  $A + (B + C) = (A + B) + C$

**3) Existence of Identity**

As it is shown in previous section that the null matrix, denoted by  $\mathbf{0}$ , falls in class CJ4B,  
i.e. For  $A \in CJ4B, \exists \mathbf{0} \in CJ4B$   
 $\exists A + \mathbf{0} = \mathbf{0} + A = A$ .  
This shows existence of additive identity in CJ4B.

**4) Existence of Additive Inverses**

In 2.1.1, we have defined multiplication of  $A \in CJ4B$  by a scalar  $c \in R$ .  
For  $c = -1$ , we have  $-1 A = -A$  which belongs to CJ4B.  
Now following regular addition activity we have  
 $A + (-A) = -A + A = \mathbf{0}$  (Additive identity)  
Accepting the above mentioned properties the set CJ4B under ‘+’ forms a group structure  
i.e.  $(CJ4B, +)$  is a group

....(18)

### 5) Commutative Group

In addition to the above properties, in our case, we have for any two matrices of class 4B, - the group CJ4B is closed under '+'. Let  $A = (a_{ij})$  and  $B = (b_{ij})$  where  $a_{ij}$  and  $b_{ij}$  are real entries.

Since  $a_{ij}, b_{ij}$ , are real entries and commutative law holds in the set R of real values, so we have  $a_{ij} + b_{ij} = b_{ij} + a_{ij} \forall i \text{ and } j$

This in turn implies that  $A + B = B + A$

This property puts the group under the class of commutative (Abelian) group under the same operation '+'.  
 $\Rightarrow$  CJ4B, '+', is a commutative group

### 2.4 Determinant Values: conditions for some standard Matrices

In this section we attempt to impose non-singularity conditions on some standard matrices of different class. We note that the standard forms may vary but the governing principles for the construction of classes remain the same.

### 2.5 Determinant Values: conditions for some standard Matrices

In this section we attempt to impose non-singularity conditions on some standard matrices of different class. We note that the standard forms may vary but the governing principles for the construction of classes remain the same.

Table 1

No	Class	Order	Standard Form	Libra Value	Determinant Value
1	CJ1	3x3	$A = \begin{bmatrix} a & c & e \\ b & d & f \\ p - (a + b) & p - (c + d) & p - (e + f) \end{bmatrix}$	$L(A) = p A $	$p \left[ \begin{matrix} (ad - bc) - \\ (af - be) + (cf - de) \end{matrix} \right]$
2	CJ2	3x3	$A = \begin{bmatrix} a & c & p - (a + b) \\ b & d & p - (c + d) \\ e & f & p - (e + f) \end{bmatrix}$	$L(A) = p A $	$p \left[ \begin{matrix} (ad - bc) - \\ (af - be) + (cf - de) \end{matrix} \right]$
3	CJ3	3x3	$A = \begin{bmatrix} a & c & p - (a + b) \\ b & d & p - (c + d) \\ p - (a + c) & p - (b + d) & p + a + b + c + d \end{bmatrix}$	$L(A) = p$	$p[5(ad - bc) + p(b + c - a - d)]$
4	CJ4B	3x3	$A = \begin{bmatrix} p + k & p - k & p + k \\ p + k & p & p \\ p - k & p + 2k & p \end{bmatrix}$	$L(A) = 3p + k$	$ A  = 2k^2(3p + k)$
5	CJ4B	3x3	$A = \begin{bmatrix} b & a & p - (a + b) \\ a & p - (a + b) & b \\ p - (a + b) & b & a \end{bmatrix}$	$L(A) = p$	$ A  = -p \left[ \begin{matrix} p^2 - 3p(a + b) \\ + 3(a^2 + ab + b^3) \end{matrix} \right]$
6	CJ4A	3x3	$A = \begin{bmatrix} p + k & p - k & p + k \\ p & p & p + k \\ p & p + 2k & p - k \end{bmatrix}$	$L(A) = 3p + k$	$ A  = -2k^2(3p + k)$
7	CJ4B	4 x 4	$A = \begin{bmatrix} p + k & -p + 2k & 2p - 4k & p + 2k \\ 2p - k & 2p - 2k & 5k & -p - k \\ -p - k & 3p + 2k & -p + k & 2p - k \\ p + 2k & -p - k & 2p - k & p + k \end{bmatrix}$	$L(A) = 3p + k$	$ A  = 4k(3p + k). \\ (5k^2 - 9kp - 8p^2)$

### 3. Symmetric Matrices

In this section we introduce symmetric matrices of class 4B and discuss some fundamental properties.

For three real values say p, k and m, we have a symmetric matrix about leading diagonal.

For different values of p, k and m the following format generalizes a sub-class of the class 4B. we denote this sub - class as 'CJS4B'.

$$A = \begin{bmatrix} p - k & p - m & p + m + 2k \\ p - m & p + m + 2k & p - k \\ p + m + 2k & p - k & p - m \end{bmatrix} \text{ with } L(A) = 3p + k \quad \dots (19)$$

Recording the matrix for

$$P = 1, k = 2, \text{ and } m = -1,$$

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 2 & 4 & -1 \\ 4 & -1 & 2 \end{bmatrix}, A \in CJS4B(3 \times 3, L(A))$$

We have this sub-class CJS4B an Abelian group under ‘+’.

As we have calculated the determinant of this matrix denoted as  $|A|$  or dot. A a real value. It is given as follows.

$$|A| = - (3p + k) (7k^2 + 6km + 3m^2) \quad \dots (20)$$

This can be verified from the illustration (19) of the previous pan group.

### 3.1 Extended CJ4B System

At this stage we extend our existing system of class 4B matrices. This extension will help able to develop some other mathematical aspects. A system defined as class 4, which is the union of two known classes, 4A and 4B.

$$CJ4 = CJ4A \cup CJ4B$$

[Note: These two classes differ only in the case of libra value (sum of all entries), the class 4A satisfies libra value property in the case of non-leading (secondary) diagonal while the class 4B holds the libra value property not in the case of non-leading diagonal but it holds for the case of principal (leading) diagonal.]

$$\text{Define } CJ4B^* = CJ4B \cup I \text{ where } I \text{ is an identity matrix.} \quad (21)$$

$$\text{Clearly } CJ4B^* \subset CJ4 = CJ4A \cup CJ4B$$

### 3.2 Extension to commutative Group:

We have already shown that the class 4B agrees to an Abelian structure. We, in the following section, prove extended version and establish more algebraic properties.

$$A = \begin{bmatrix} p+k & p-k & p+k \\ p+k & p & p \\ p-k & p+2k & p \end{bmatrix}$$

Let us consider the format

$$A \in CJ4B^* (3 \times 3, L(A) = 3p+k)$$

In addition to what we have already established we will attempt to prove that the class 4B of the matrices in the above format forms a field under both operations ‘+’ and ‘.’

**Rest part of this manuscript is continued in Part B.....**