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Some subclasses of q -spirallike meromorphic functions

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Abstract
In this paper, we introduce and investigate two new subclasses $\Sigma\mathcal{S}(\lambda,\beta,q)$ and $\Sigma\mathcal{C}(\lambda,\beta,q)$ of q -spirallike meromorphic functions defined in the punctured open unit disc.

Keywords: univalent functions, meromorphic functions, meromorphic q -spirallike

1. Introduction

Let Σ denote the class of functions f of the form

$$f(z)=\frac{1}{z}+\sum_{n=0}^{\infty}a_n z^n, \tag{1}$$

Which are analytic in the open disc $\mathcal{U}=\{z: z\in\mathbb{C} and 0<|z|<1\}$. Let $\mathcal{S}$ denote the subclass of function in Σ which are univalent in $\mathcal{U}$. A functions $f\in\Sigma$ is said to be meromorphic starlike of order $\beta(0\le\beta<1)$ if it satisfies

$$-\Re\left\{\frac{zf'(z)}{f(z)}\right\}>\beta(z\in\mathcal{U},0\le\beta<1), \tag{2}$$

And we denote this class by $\Sigma\mathcal{S}(\beta)$.
If $f\in\Sigma$ satisfies

$$-\Re\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\beta(z\in\mathcal{U},0\le\beta<1), \tag{3}$$

We say that f is meromorphic convex of order β and we denote this class by $\Sigma\mathcal{C}(\beta)$. In [7] Jackson introduced and studied the concept of the q -derivative operator ∂_q as follows

$$\partial_q f(z)=\frac{f(z)-f(qz)}{z(1-q)},(z\neq 0,0<q<1,\partial_q f(0)=f'(0)). \tag{4}$$

Equivalently (4) may be written as

$$\partial_q f(z)=1+\sum_{n=2}^{\infty}[n]_q a_n z^{n-1}, z\neq 0, \tag{5}$$

Where $[n]_q=\frac{1-q^n}{1-q}$,, note that as $q\to 1^-,[n]_q\to n$.

Definition 1.1 *A function $f\in\Sigma$ is said to be meromorphic q -spirallike of order β if it satisfies*

$$-\Re\left\{e^{i\lambda}\frac{z\partial_q f(z)}{f(z)}\right\}>\beta(z\in\mathcal{U},0\le\beta<1), \tag{6}$$

We denote this class by $\Sigma\mathcal{S}(\lambda,\beta,q)$.

Definition 1.2 *A function $f\in\Sigma$ is said to be meromorphic convex q -spirallike of order β if it satisfies*

$$-\Re\left\{e^{i\lambda}\frac{\partial_q(z\partial_q f(z))}{\partial_q f(z)}\right\}>\beta(z\in\mathcal{U},0\le\beta<1), \tag{7}$$

We denote this class by $\Sigma\mathcal{C}(\lambda,\beta,q)$.

Let $\Sigma\mathcal{S}(\lambda, A, B, q)$ and $\Sigma\mathcal{C}(\lambda, A, B, q)$, $-1 \leq A < B \leq 1$ be the subclasses of functions in Σ satisfying

$$-\left\{e^{i\lambda}\frac{z\partial_q f(z)}{f(z)}\right\} \prec \frac{1+Az}{1+Bz}, (|\lambda| < \frac{\pi}{2}, z \in \mathcal{U}, \tag{8}$$

$$-\left\{e^{i\lambda}\frac{\partial_q(z\partial_q f(z))}{\partial_q f(z)}\right\} \prec \frac{1+Az}{1+Bz}, (|\lambda| < \frac{\pi}{2}, z \in \mathcal{U}. \tag{9}$$

2. Main results

Definition 2.1 *Let* $H(z) = \frac{(z\partial_q f(z))}{f(z)}$ *for* $f(z) \in \mathcal{S}$. *A function* $f(z)$ *is said to be in the class* $\Sigma\mathcal{S}(\lambda, \beta, q)$ *if it satisfies*

$$\left|\frac{1}{e^{i\lambda}H(z)} + \frac{1}{2\beta}\right| < \frac{1}{2\beta} (z \in \mathcal{U}), \tag{10}$$

For some real number $0 < \beta < 1, 0 < q < 1$ and $|\lambda| < \frac{\pi}{2}$.

Theorem 2.1 $f(z) \in \Sigma\mathcal{S}(\lambda, \beta, q)$ *if and only if,* $\Re\left(e^{i\lambda}\frac{z\partial_q f(z)}{f(z)}\right) < -\beta$.

Proof. Let $H(z) = \frac{z\partial_q f(z)}{f(z)}$ for $f \in \mathcal{S}$. If $f(z) \in \Sigma\mathcal{S}(\lambda, \beta, q)$, we can write

$$\left|\frac{2\beta + e^{i\lambda}H(z)}{2\beta e^{i\lambda}H(z)}\right| < \frac{1}{2\beta}.$$

Then, we can get

$$\left|\frac{2\beta + e^{i\lambda}H(z)}{2\beta e^{i\lambda}H(z)}\right| < \frac{1}{2\beta} \Leftrightarrow \left|\frac{2\beta + e^{i\lambda}H(z)}{2\beta e^{i\lambda}H(z)}\right|^2 < \left(\frac{1}{2\beta}\right)^2$$

$$\Leftrightarrow (2\beta + e^{i\lambda}H(z))\left[\overline{2\beta + e^{i\lambda}H(z)}\right] < \left[\overline{e^{i\lambda}H(z)}\right] e^{i\lambda}H(z)$$

$$\Leftrightarrow (2\beta + e^{i\lambda}H(z))\left[\overline{2\beta + e^{i\lambda}H(z)}\right] < \left[e^{-i\lambda}\overline{H(z)}\right] e^{i\lambda}H(z)$$

$$\Leftrightarrow 4\beta^2 + 2\beta e^{-i\lambda}\overline{H(z)} + 2\beta e^{-i\lambda}H(z) + H(z)\overline{H(z)} < H(z)\overline{H(z)}$$

$$\Leftrightarrow 4\beta^2 + 2\beta\left(e^{-i\lambda}\overline{H(z)} + e^{i\lambda}H(z)\right) < 0$$

$$\Leftrightarrow 2\beta + 2\Re(e^{i\lambda}H(z) < 0$$

$$\Leftrightarrow \Re(e^{i\lambda}H(z) < -\beta$$

$$\Leftrightarrow \Re\left(e^{i\lambda}\frac{z\partial_q f(z)}{f(z)}\right) < -\beta.$$

As $q \to 1^-$, we get the following result proved by Alamoush [1].

Corollary 2.1 $f(z) \in \Sigma\mathcal{S}(\lambda, \beta)$ *if and only if,* $\Re\left(e^{i\lambda}\frac{zf(z)}{f(z)}\right) < -\beta$.

Theorem 2.2 *If* $f(z) \in \Sigma$ *satisfies*

$$\sum_{n=2}^{\infty} \left\{[n]_q + |[n]_q + 2\beta e^{-i\lambda}||a_n|\right\} \leq 1 - |-1 + 2q\beta e^{-i\lambda}|, \tag{11}$$

For some $|\lambda| < \frac{\pi}{2}, 0 < q < 1$ and $0 < \beta < \cos\lambda$, then $f(z) \in \Sigma\mathcal{S}(\lambda, \beta, q)$.

Proof. It suffices to show that

$$\left|\frac{2\beta + e^{i\lambda}H(z)}{e^{i\lambda}H(z)}\right| < 1,$$

For some $|\lambda| < \frac{\pi}{2}, 0 < q < 1$ and $0 < \beta < \cos\lambda$, where $H(z) = \frac{z\partial_q f(z)}{f(z)}$. Note that

$$\left|\frac{2\beta+e^{i\lambda}H(z)}{e^{i\lambda}H(z)}\right| = \left|\frac{2\beta f(z)+e^{i\lambda}z\partial_q f(z)}{e^{i\lambda}z\partial_q f(z)}\right|$$
$$= \left|\frac{2\beta e^{-i\lambda}f(z)+z\partial_q f(z)}{z\partial_q f(z)}\right|$$
$$= \left|\frac{(-1+2q\beta e^{-i\lambda})+q\sum_{n=2}^{\infty}([n]_q-2\beta e^{-i\lambda})|a_n|z^{n+1}}{-1+q\sum_{n=2}^{\infty}[n]_q a_n z^{n+1}}\right|$$
$$\leq \frac{|-1+2q\beta e^{-i\lambda}|+q\sum_{n=2}^{\infty}|[n]_q-2\beta e^{-i\lambda}||a_n||z|^{n+1}}{1-q\sum_{n=2}^{\infty}[n]_q|a_n||z|^{n+1}}$$

$$< \frac{|-1+2q\beta e^{-i\lambda}|+q\sum_{n=2}^{\infty}|[n]_q-2\beta e^{-i\lambda}||a_n|}{1-q\sum_{n=2}^{\infty}[n]_q|a_n|} < 1. \tag{12}$$

Therefore, if
$q\sum_{n=2}^{\infty}\{[n]_q+|[n]_q+2\beta e^{-i\lambda}||a_n|\}\leq 1-|-1+2q\beta e^{-i\lambda}|$,
For some $|\lambda|<\frac{\pi}{2}, 0<q<1$ and $0<\beta<\cos\lambda$, then
$q\sum_{n=2}^{\infty}|[n]_q-2\beta e^{-i\lambda}||a_n|\leq 1-|[-1+2q\beta e^{-i\lambda}|-q\sum_{n=2}^{\infty}[n]_q|a_n|$.
Using this inequality in (9), we get

$$\left|\frac{2\beta-e^{i\lambda}H(z)}{e^{i\lambda}H(z)}\right| < \frac{|-1+2q\beta e^{-i\lambda}|+q\sum_{n=2}^{\infty}|[n]_q-2\beta e^{-i\lambda}||a_n|}{1-q\sum_{n=2}^{\infty}[n]_q|a_n|}$$
$$\leq \frac{|-1+2q\beta e^{-i\lambda}|+1-|-1+2q\beta e^{-i\lambda}|-q\sum_{n=2}^{\infty}[n]_q|a_n|}{1-q\sum_{n=2}^{\infty}[n]_q|a_n|}$$
$$=1.$$

Therefore, $f(z)\in\mathcal{S}(\lambda,\beta,q)$, for some $|\lambda|<\frac{\pi}{2}, 0<q<1$ and $0<\beta<\cos\lambda$.
As $q\to 1^-$, we get the following result proved by Alamoush [1].

Corollary 2.2 *If* $f(z)\in\Sigma$ *satisfies*

$$\sum_{n=2}^{\infty}\{n+|n+2\beta e^{-i\lambda}||a_n|\}\leq 1-|-1+2\beta e^{-i\lambda}|, \tag{13}$$

For some $|\lambda|<\frac{\pi}{2}$ and $0<\beta<\cos\lambda$, then $f(z)\in\Sigma\mathcal{S}(\lambda,\beta)$.

Taking $\lambda=\frac{\pi}{4}$ in Theorem 2.2, we get the following Theorem

Theorem 2.3 *If* $f(z)\in\Sigma$ *satisfies*

$$q\sum_{n=2}^{\infty}\{[n]_q+|[n]_q+2\beta||a_n|\}\leq 1-|-1+2q\beta|, \tag{14}$$

For some $0<\beta<\cos\lambda$ and $0<q<1$, then $f(z)\in\Sigma\mathcal{S}(0,\beta,q)$.

Definition 2.2 *Let* $H_q(z)=\left\{1+\frac{qz\partial_q^2 f(z)}{\partial_q f(z)}\right\}$ *for* $f(z)\in\mathcal{S}$. *A function* $f(z)$ *is said to Be in the class* $\Sigma\mathcal{C}(\lambda,\beta,q)$ *i if it satisfies*

$$\left|\frac{1}{e^{i\lambda}H_q(z)}+\frac{1}{2\beta}\right|<\frac{1}{2\beta}(z\in\mathcal{U}), \tag{15}$$

For some real number $0<\beta<1, 0<q<1$ and $|\lambda|<\frac{\pi}{2}$..

Theorem 2.4 $f(z)\in\Sigma\mathcal{C}(\lambda,\beta,q)$ *if and only if,* $\Re\left\{e^{i\lambda}\left(1+\frac{qz\partial_q^2 f(z)}{\partial_q f(z)}\right)\right\}<-\beta$..

Proof. Let $H_q(z)=\left\{1+\frac{qz\partial_q f(z)}{\partial_q f(z)}\right\}$ for $f(z)\in\mathcal{S}$. If $f\in\mathcal{C}(\lambda,\beta,q)$, we can write

$$\left|\frac{2\beta+e^{i\lambda}H_q(z)}{2\beta e^{i\lambda}H_q(z)}\right|<\frac{1}{2\beta}.$$

Then, we can get

$$\left|\frac{2\beta+e^{i\lambda}H_q(z)}{2\beta\, e^{i\lambda}H_q(z)}\right| < \frac{1}{2\beta} \Leftrightarrow \left|\frac{2\beta+e^{i\lambda}H_q(z)}{2\beta\, e^{i\lambda}H_q(z)}\right|^2 < \left(\frac{1}{2\beta}\right)^2.$$

$$\Leftrightarrow \left|\frac{2\beta-e^{i\lambda}H_q(z)}{2\beta\, e^{i\lambda}H_q(z)}\right|^2 < \left(\frac{1}{2\beta}\right)^2$$

$$\Leftrightarrow (2\beta + e^{i\lambda}H_q(z))\left[\overline{2\beta + e^{i\lambda}H_q(z)}\right] < \left[\overline{e^{i\lambda}H_q(z)}\right]e^{i\lambda}H_q(z)$$

$$\Leftrightarrow (2\beta + e^{i\lambda}H_q(z))\left[\overline{2\beta + e^{i\lambda}H_q(z)}\right] < \left[e^{-i\lambda}\overline{H_q(z)}\right]e^{i\lambda}H_q(z)$$

$$\Leftrightarrow 4\beta^2 + 2\beta e^{-i\lambda}\overline{H_q(z)} + 2\beta e^{-i\lambda}H_q(z) + H_q(z)\overline{H_q(z)} < H_q(z)\overline{H_q(z)}$$

$$\Leftrightarrow 4\beta^2 + 2\beta\left(e^{-i\lambda}\overline{H_q(z)} + e^{i\lambda}H_q(z)\right) < 0$$

$$\Leftrightarrow 2\beta + 2\Re(e^{i\lambda}H_q(z) < 0$$

$$\Leftrightarrow \Re(e^{i\lambda}H_q(z) < -\beta$$

$$\Leftrightarrow \Re\left\{e^{i\lambda}\left(1 + \frac{qz\partial_q^2 f(z)}{\partial_q f(z)}\right)\right\} < -\beta.$$

As $q \to 1^-$, we get the following result proved by Alamoush [1].

Corollary 2.3 $f(z) \in \Sigma\mathcal{C}(\lambda,\beta)$ *if and only if,* $\Re\left\{e^{i\lambda}\left(1 + \frac{zf''(z)}{f'(z)}\right)\right\} < -\beta.$

Theorem 2.5 *If* $f(z) \in \Sigma$ *satisfies*

$$q\sum_{n=2}^{\infty}[n]_q\left((1+q[n-1]_q) + |2\beta e^{-i\lambda} + 1 + q[n-1]_q||a_n|\right) \leq -([1+ \quad q|2\beta e^{-i\lambda}+1|), \tag{16}$$

For some $|\lambda| < \frac{\pi}{2}, 0 < q < 1$ and $0 < \beta < \cos\lambda$, then $f(z) \in \mathcal{C}(\lambda,\beta,q)$.

Proof. It suffices to show that

$$\left|\frac{2\beta+e^{i\lambda}H_q(z)}{e^{i\lambda}H_q(z)}\right| < 1,$$

For some $|\lambda| < \frac{\pi}{2}, 0 < q < 1$ and $0 < \beta < \cos\lambda$, where $H_q(z) = \left\{1 + \frac{qz\partial_q^2 f(z)}{\partial_q f(z)}\right\}$.. Note that

$$\left|\frac{2\beta-e^{i\lambda}H_q(z)}{e^{i\lambda}H_q(z)}\right| = \left|\frac{2\beta e^{-i\lambda}\partial_q f(z)+zq\partial_q^2 f(z)}{\partial_q f(z)+qz\partial_q^2 f(z)}\right|$$

$$= \left|\frac{(2\beta e^{-i\lambda}+1)\partial_q f(z)+qz\partial_q^2 f(z)}{\partial_q f(z)+qz\partial_q^2 f(z)}\right|$$

$$= \left|\frac{(2\beta e^{-i\lambda}+1)(\frac{-1}{qz^2}+\sum_{n=2}^{\infty}[n]_q a_n z^{n-1}) + \frac{[2]q}{qz^2} + q\sum_{n=2}^{\infty}[n-1]_q[n]_q a_n z^{n-1}}{(\frac{-1}{qz^2}+\sum_{n=2}^{\infty}[n]_q a_n z^{n-1}) + \frac{[2]q}{qz^2} + q\sum_{n=2}^{\infty}[n-1]_q[n]_q a_n z^{n-1}}\right|$$

$$= \left|\frac{-(2\beta e^{-i\lambda}+1)+q\sum_{n=2}^{\infty}[n]_q(2\beta e^{-i\lambda}+1+q[n-1]_q)a_n z^{n+1}+[2]_q}{([2]_q-[1]_q)+q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)a_n z^{n+1}}\right|$$

$$< \frac{|2\beta e^{-i\lambda}+1|+q\sum_{n=2}^{\infty}[n]_q|2\beta e^{-i\lambda}+1+q[n-1]_q||a_n|+[2]_q}{([2]_q-1)-q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)|a_n|} < 1. \tag{17}$$

Therefore, if $q\sum_{n=2}^{\infty}[n]_q\left((1+q[n-1]_q) + |2\beta e^{-i\lambda} + 1 + q[n-1]_q||a_n|\right) \leq -(1+|2\beta e^{-i\lambda}+1|)$,
For some $|\lambda| < \frac{\pi}{2}$, and $0 < \beta < \cos\lambda$, then

$$q\sum_{n=2}^{\infty}[n]_q|2\beta e^{-i\lambda} + 1 + q[n-1]_q||a_n| \leq -1 - |2\beta e^{-i\lambda}+1| - q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)|a_n|.$$

Using this inequality in (14), we get

$$\left|\frac{2\beta+e^{i\lambda}H_q(z)}{e^{i\lambda}H_q(z)}\right| < \frac{|2\beta e^{-i\lambda}+1|+q\sum_{n=2}^{\infty}[n]_q|2\beta e^{-i\lambda}+1+q[n-1]_q||a_n|+[2]_q}{([2]_q-1)-q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)|a_n|}$$

$$\leq \frac{|2\beta e^{-i\lambda}+1|-1-|2\beta e^{-i\lambda}+1|-q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)|a_n|+[2]_q}{([2]_q-1)-q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)|a_n|}$$

$$= \frac{([2]_q-1)-q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)|a_n|}{([2]_q-1)-q\sum_{n=2}^{\infty}[n]_q(1+q[n-1]_q)|a_n|} = 1.$$

Therefore $f(z) \in \Sigma\mathcal{C}(\lambda, \beta, q)$ for some $|\lambda| < \frac{\pi}{2}, 0 < q < 1$ and $0 < \beta < \cos\lambda$.
As $q \to 1^-$, we get the following result proved by Alamoush [1].

Corollary 2.4 *If* $f(z) \in \Sigma$ *satisfies*

$$\sum_{n=2}^{\infty} n\left(n + |2\beta e^{-i\lambda} + n||a_n|\right) \leq -(1 + |2\beta e^{-i\lambda} + 1|), \tag{18}$$

For some $|\lambda| < \frac{\pi}{2}$, and $0 < \beta < \cos\lambda$, then $f(z) \in \mathcal{C}(\lambda, \beta)$.

Theorem 2.6 *If* $f(z) \in \Sigma$ *satisfies*

$$q\sum_{n=2}^{\infty} \left\{|1 + [n]_q e^{i\lambda}| + |A + [n]_q Be^{i\lambda}|\right\}|a_n| \leq |qA - Be^{i\lambda}| - |q - e^{i\lambda}|, \tag{19}$$

For some $|\lambda| < \frac{\pi}{2}$ and$0 < q < 1$, then
$f(z) \in \Sigma\mathcal{S}(\lambda, A, B, q)$.
Proof. It is suffices to show that

$$\left|\frac{1 + e^{i\lambda}\frac{z\partial_q f(z)}{f(z)}}{A + Be^{i\lambda}\frac{z\partial_q f(z)}{f(z)}}\right| < 1,$$

That is

$$\left|\frac{f(z) + e^{i\lambda}z\partial_q(z)}{Af(z) + Be^{i\lambda}z\partial_q(z)}\right| < 1.$$

Let

$$\left|\frac{f(z) + e^{i\lambda}z\partial_q(z)}{Af(z) + Be^{i\lambda}z\partial_q(z)}\right| = \left|\frac{\frac{1}{z} + \sum_{n=2}^{\infty} a_n z^n + e^{i\lambda}\left(\frac{-1}{qz} + \sum_{n=2}^{\infty}[n]_q a_n z^n +\right)}{A\left(\frac{1}{z} + \sum_{n=2}^{\infty} a_n z^n\right) + Be^{i\lambda}\left(\frac{-1}{qz} + \sum_{n=2}^{\infty}[n]_q a_n z^n\right)}\right|$$

$$= \left|\frac{q + q\sum_{n=2}^{\infty} a_n z^{n-1} - e^{i\lambda} + q\sum_{n=2}^{\infty} e^{i\lambda}[n]_q a_n z^{n-1}}{qA + qA\sum_{n=2}^{\infty} a_n z^{n-1} - Be^{i\lambda} + q\sum_{n=2}^{\infty} Be^{i\lambda}[n]_q a_n z^{n-1}}\right|$$

$$\leq \frac{|q - e^{i\lambda}| + q\sum_{n=2}^{\infty}|1 + [n]_q e^{i\lambda}||a_n|}{|qA - Be^{i\lambda}| - q\sum_{n=2}^{\infty}|A + [n]_q Be^{i\lambda}||a_n|}. \tag{20}$$

Hence (17) is bounded by 1, if

$$q\sum_{n=2}^{\infty} |1 + [n]_q e^{i\lambda}| + |A + [n]_q Be^{i\lambda}||a_n| \leq |qA - Be^{i\lambda}| - |q - e^{i\lambda}|.$$

As $q \to 1^-$, we get the following result proved by Alamoush [1].

Corollary 2.5 *If* $f(z) \in \Sigma$ *satisfies*

$$\sum_{n=2}^{\infty} \left\{|1 + ne^{i\lambda}| + |A + nBe^{i\lambda}|\right\}|a_n| \leq |A - Be^{i\lambda}| - |1 - e^{i\lambda}|, \tag{21}$$

For some $|\lambda| < \frac{\pi}{2}$ and, then
$f(z) \in \Sigma\mathcal{S}(\lambda, A, B)$.

Theorem 2.7 *If* $f(z) \in \Sigma$ *satisfies*

$$q\sum_{n=2}^{\infty} [n]_q\left\{|2 + q[n-1]_q e^{i\lambda}| + |A + 1 + q[n-1]_q Be^{i\lambda}|\right\}|a_n| \leq |qA - Be^{i\lambda}| - |q - e^{i\lambda}| \tag{22}$$

For some $|\lambda| < \frac{\pi}{2}$, then
$f(z) \in \Sigma\mathcal{C}(\lambda, A, B, q)$.

As $q \to 1^-$, we get the following result proved by Alamoush [1].

Corollary 2.6 *If* $f(z) \in \Sigma$ *satisfies*

$$\sum_{n=2}^{\infty} n\left\{|1 + ne^{i\lambda}| + |A + nBe^{i\lambda}|\right\}|a_n| \leq |A - Be^{i\lambda}| - |1 - e^{i\lambda}|, \tag{23}$$

For some $|\lambda| < \frac{\pi}{2}$ and, then
$f(z) \in \Sigma C(\lambda, A, B)$.

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