International Journal of Advanced Science and Research
www.allsciencejournal.com
ISSN: 2455-4227
Received: 30-10-2022, Accepted: 16-11-2022, Published: 02-12-2022
Volume 7, Issue 2, 2022, Page No. 51-63

MHD stagnation point flow of a williamson nanofluid over a unsteady stretching sheet with convective boundary conditions and nonuniform heat source/sink

Ch. Kishore Kumar, S Renuka
Department of Mathematics, Nizam College, Osmania University, Hyderabad, Telangana, India

Abstract
The main emphasis of the present study is to analyze the MHD stagnation point flow of a Williamson nanofluid over a unsteady stretching sheet with convective boundary conditions and non-uniform heat source/sink. Using suitable similarity transformations the governing partial differential equations were transformed into coupled nonlinear ordinary differential equations with. These equations were then solved by numerical technique known as Runge-Kutta-Felhberg 45 order method. Obtained numerical solutions are presented in the form of graphs and tables for different values of flow pertinent parameters such as; magnetic parameter(M), Prandtl number (Pr), velocity ratio parameter (λ), Williamson parameter (W), Lewis number (Le), Unsteadiness parameter (A), Brownian motion parameter (Nb), thermophoresis parameter (Nt), Biot number (δ) and non-uniform heat source/sink (A*, B*). Numerical results are obtained for velocity, temperature and concentration, as well as the skin friction coefficient, local Nusselt number and local Sherwood number.

Keywords: velocity ratio parameter, williamson parameter, Stagnation point, unsteadiness parameter, brownian motion and thermophoresis parameters, non-uniform heat source/sink

Introduction

The classical study of viscous stagnation point flow towards a rigid horizontal plane and a rigid horizontal axi-symmetric surface is quite interesting study as they admit the exact solutions for the Navier–Stokes equations. The study of stagnation point flow over a stretching sheet was first done by Hiemenz [1] and he analyzed exactly by the Navier–Stokes equations and reported on the velocity distribution for the two-dimensional case. Therefore this plane study is referred as Hiemanz flow and this work was extended by Howarth [2]. The same study was extended for three dimensional case by Gersten *et al.* [3]. Based on the study of Weidman and Putkaradze (2003), these flows were characterized as inviscid or viscous, steady or unsteady, two-dimensional or three-dimensional, symmetric or asymmetric, normal or oblique, homogeneous or two-fluid, and forward or reverse. The study on heat and mass transfer flow over a stretching sheet was attracting the young researchers because of its wide applications. The first study viscous fluid caused by a stretching sheet was done by Crane [4] for the viscous fluid. This study was extended by Mc Cormack and Crane [5], they provided a comprehensive discussion on boundary layer flow caused by stretching of an elastic flat sheet moving in its own plane with velocity varying linearly with distance. Later the MHD flows over a stretching by considering variations parameters into considerations given in the literature [6-10].

Stagnation point is a point where the fluid velocity of the local particle is zero. Chiam [11] investigated the stagnation-point flow towards a stretching sheet and found no boundary layer structure near the sheet. This work is reinvestigated by Mahapatra and Gupta [12] for the same stagnation-point flow towards a stretching sheet and found two kinds of boundary layer near the sheet depending on the ratio of the stretching and straining rates. Ashraf [13] incorporated the heat transfer parameter to stagnation point flow and studied MHD stagnation point flow of a micropolar fluid towards a heated vertical surface.

All the above investigations dealt with the steady state flow. However, in some cases, the flow field, heat and mass transfer can be unsteady due to a sudden stretching of the flat sheet or by a step change of the temperature of the sheet. The unsteady flow over a stretching sheet was first done by Pop [14] where as Wang [15] investigated on the Impulsive stretching of a surface in a viscous fluid.. The unsteady Mixed convection flow over a porous stretching sheet was done by Elbashbeshy and Aldawody [16]. Later the same study was extended by considering various fluids like the Casson fluid flow over an unsteady stretching sheet was studied by Swathi Mukhopadhyay *et al* [17]. The investigations on nano fluids and dusty fluid over unsteady stretching sheet was done by Kalidas *et al*. [18] and B.J. Gireesha [19]. Whereas the HAM solutions for unsteady boundary-layer flows caused by an impulsively stretching plate was investigated by Shijun Lio [20] and gave a compared result with Pade's Approximation.

The term Nanofluid was first introduced by Choi [21]. It refers to a liquid containing a dispersion of submicronic solid particles (nanoparticles) with typical length on the order of 1–50 nm. and it has a characteristic feature of thermal conductivity enhancement this was observed by Masuda *et al*. [22] Furthermore, Eastman *et al*. [23] has conducted an experimental study on thermal conductivity enhancement of ethylene glycol-based nanofluids containing copper nanoparticles. And then the investigations on nano fluid was later extended to the natural convective boundary-layer flow over vertical plate by Kuznetsov and Nield [24]. In addition to this the flow over a stretching sheet has attracting the researchers because of its applications and novelty. So in the recent years the study on flow of a nanofluid over stretching sheet takes place In view of this the W.A. Khan and I. Pop [25] has used a model for the nanofluid incorporates the effects of Brownian motion and thermophoresis and obtained the similarity solution for the boundary layer flow of nanofluid past a stretching sheet. Furthermore the analytical solution for the same problem was obtained by Hassani *et al* [26]. Wubshet *et al* [27] investigated on MHD stagnation point over stretching sheet. Whereas the slip flow effects of nanofluid over a stretching sheet was investigated by Wubshet and Shanker [28]. Moreover Nadeem and Rizwan Ul Haq [29] investigated the Effect of Thermal Radiation for Megnetohydrodynamic Boundary Layer Flow of a Nanofluid Past a Stretching Sheet with Convective Boundary Conditions.

The non-Newtonian fluids has wide range of applications in engineering and industrial processes, such as food mixing and chyme movement in the intestine, flow of blood, flow of plasma, flow of mercury amalgams and lubrications with heavy oils and greases…..Non-Newtonian fluids are grouped property-wise, as visco-inelastic fluids, visco-elastic fluids, polar fluids, anisotropic fluids and fluids with micro-structure. Out of these Williamson fluid is one of visco-inelastic fluids. Although nanofluids have been largely studied as Newtonian fluid, very recently, their rheological properties are established by non-Newtonian modeling of nanofluid transport phenomena. Many studies are focused on non-Newtonian fluid as a base fluid with suspended nanoparticles over a stretching sheet…Alam Khan and Khan [30] obtained the series solution by employing homotopy analysis method (HAM) in four flow problems of a Williamson fluid, namely Blasius, Sakiadis, stretching and stagnation point flows. Nadeem *et al*. [31] presented the modeling of a two-dimensional flow analysis for Williamson fluid over a linear and exponentially stretching surface.

Motivated by the above studies we would like to consider the effects MHD stagnation point flow of a Williamson nanofluid over an unsteady stretching sheet with convective boundary conditions of non uniform heat source of Williamson fluid over a stretching sheet. Up to my knowledge still no one has consider this study. Therefore, this paper can be used as a bridge to fill the knowledge gap

Mathematical formulation

Consider a two-dimensional flow of an incompressible Williamson nanofluid over an unsteady stretching sheet. The horizontal sheet is placed at $y = 0$ and the flow is confined in the region $y > 0$. The sheet is stretched by two equal and opposite forces. The x-axis is taken along the stretching surface in the direction of the motion with the slot as the origin, and the y-axis is perpendicular to the sheet in the outward direction towards the fluid. The flow is subjected to the effect of magnetic field of strength B_0 which is applied normal to the surface. The induced magnetic field is also assumed to be small compared to the applied magnetic field; so it is neglected

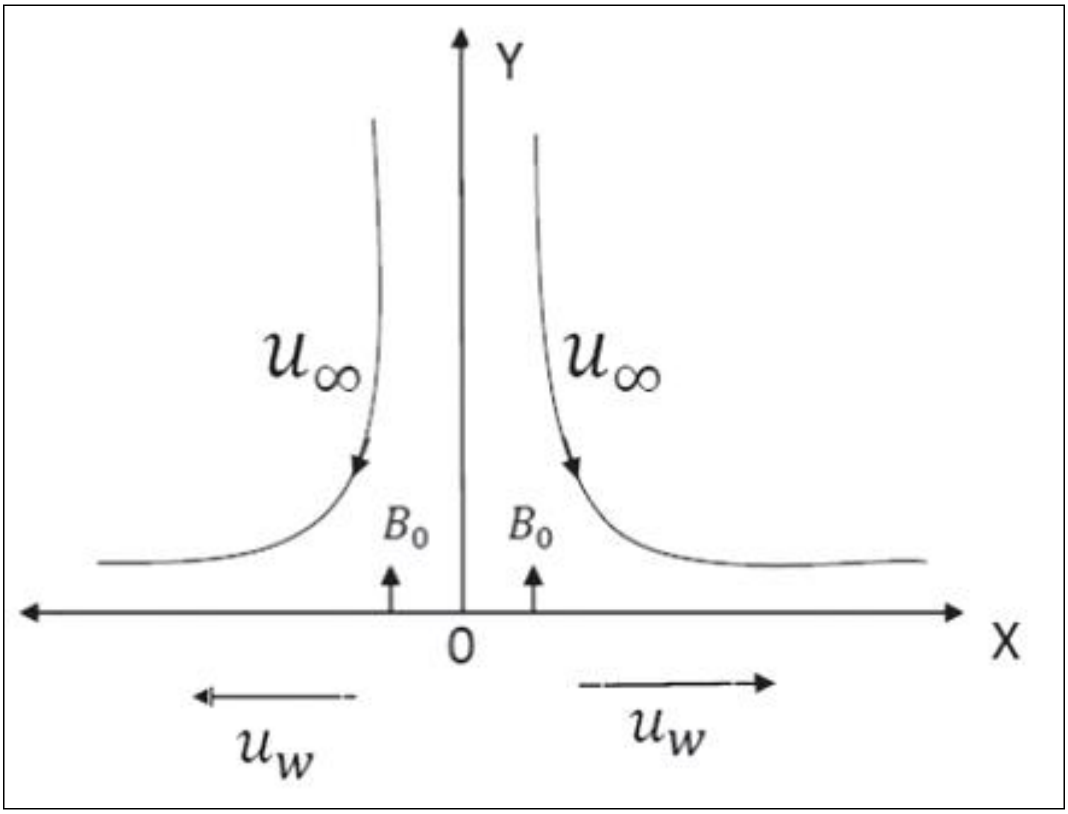

Fig 1: Physical sketch of the given problem

Details of the Williamson fluid model can be found in Nadeem [39]. For the present fluid model, the Cauchy stress tensor S is defined as;

$$S = -pI + \tau$$

$$\tau = \left(\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma\gamma}\right) A_1$$

where p is the pressure, I is the identity vector, τ is the extra stress tensor, μ_0 , μ_∞ are the limiting viscosities at zero and infinite shear rates, Γ > 0 is a time constant, A_1 is the first Rivlin-Erickson tensor, and $\dot{\gamma}$ is defined as

$$\gamma = \sqrt{\frac{\pi}{2}}, \quad \pi = trace(A_1^2)$$

where π is the second invariant strain tensor. Here we have only considered the case for which

$\mu_\infty = 0$ and < 1.

Then, we obtain $\tau = \left(\frac{\mu_0}{1-\Gamma\gamma}\right) A_1$ or $\tau = \mu_0(1 + \Gamma\gamma)A_1$

Under the above assumptions, the governing equations of the conservation of mass, momentum, energy and concentration past a stretching sheet can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + U_\infty\frac{\partial U_\infty}{\partial x} + \sqrt{2}\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}(u - U_\infty) \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left(D_B\frac{\partial C}{\partial t}\frac{\partial T}{\partial t} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{q'''}{\rho C_p} \tag{3}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2 \tag{4}$$

where x and y denotes the cartesian coordinates along and normal to the sheet, respectively, *u*, *v* are the velocity components in *x*, *y* direction respectively. T is temperature, k is nanofluid thermal conductivity, D_B is Brownian diffusion coefficient, C is nanoparticle volumetric fraction, D_T is thermophoretic diffusion coefficient and T_∞ is the ambient fluid temperature, Γ > 0 is the characteristic time, ν is the kinematic viscosity, ρ is the viscosity, α is the thermal diffusivity of the fluid, C_p is the specific heat, $U_w = \frac{cx}{(1-\gamma t)}$ is the stretching velocity of the sheet, $U_\infty = \frac{ax}{(1-\gamma t)}$ is the ambient velocity.

The corresponding boundary conditions are

$$t < 0 : u = v = 0, \quad T = T_\infty, C = C_\infty \quad for\ all\ x, y$$

$$t \geq 0 \ u = u_w, v = 0, -k\frac{\partial T}{\partial y} = h_f(T_f - T)$$

$$D_B\frac{\partial C}{\partial y} + \frac{D_B}{T_\infty}\frac{\partial T}{\partial y} = 0 \quad at\ y = 0$$

$$U \to U_\infty, \quad T \to T_\infty, \quad C \to C_\infty\ as\ y \to \infty$$

By using the following suitable similarity transformations

$$\psi = \sqrt{\frac{c\nu}{1-\gamma t}}\,x f(\eta)\ \eta = y\sqrt{\frac{c\nu}{\nu(1-\gamma t)}}, \qquad u = \frac{cx}{1-\gamma t}f'(\eta), \quad v = -f\sqrt{\frac{c\nu}{(1-\gamma t)}}$$

The governing partial differential equations are transformed into following ordinary coupled nonlinear differential equations as

$$f''' + ff'' - f'^2 - A\left(\frac{\eta}{2}f'' + f'\right) - A\lambda + \lambda^2 + \mathrm{W}f''f''' + M(1 - f') = 0 \tag{5}$$

$$\frac{1}{Pr}\theta'' - A\frac{\eta}{2}\theta' + f\theta' + Nb\phi'\theta' + \frac{1}{Pr}[A^*f' + B^*\theta] = 0 \qquad (6)$$

$$\phi'' + Lef\phi' - LeA\frac{\eta}{2}\phi' + \frac{Nt}{Nb}\theta'' = 0 \qquad (7)$$

and the boundary conditions are transformed into

$$f(0) = 0 \; f'(0) = 1 \; \theta'(0) = \delta(\theta(0) - 1)$$

$$Nb\phi'(0) + Nt\theta'(0) = 0 \; at \; \eta = 0$$

$$f'(\eta) \to \lambda, \theta(\eta) \to 0, \phi(\eta) \to 0 \; as \; \eta \to \infty$$

$$A = \frac{\gamma}{c}, \lambda = \frac{a}{c}, W = \sqrt{2}\Gamma x\sqrt{\frac{c^3}{v(1-\gamma t)^3}}, Nt = \frac{\rho C_p D_T(T_f - T_\infty)C_w}{\rho c_f v}, \; Nb = \frac{\rho C_p D_B C_w}{\rho c_f v}, \; Le = \frac{v}{D_B}$$

where A, λ, W, Nb, Nt and Le denote the unsteadiness parameter, velocity ratio parameter, Williamson nanofluid parameter, Brownian motion, thermophoresis parameter and Lewis number.

The skin friction coefficient C_f, the local Nusselt number Nu_x and the Local Sherwood number Sh_x at the stretching surface defined as

$$C_f = \frac{\tau_w}{\rho U_w^2} \; Nu_x = \frac{xq_w}{k(T_f - T_\infty)}, \; Sh_x = \frac{x m_w}{D_B(C_w - C_\infty)}$$

Where τ_w and q_w are the shear stress along the sheet and the heat flux from the surface and m_w is the mass flux from the sheet. which are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right)_{y=0}, \; q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \; m_w = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$Re_x^{\frac{1}{2}}C_f = \left(f'' + \frac{\lambda}{2}f''^2\right)_{y=0}, \; Re_x^{\frac{1}{2}}Nu_x = -\theta'(0), \; \frac{Sh_x}{(Re_x)^{1/2}} = -\varphi'(0)$$

where $Re_x^{\frac{1}{2}} = \frac{U_w x}{v}$ is local Reynolds number.

Results and Discussion

The numerical solutions are obtained by adopting numerical scheme known as Runge-Kutta-Fehlberg-45 order method. To investigate influences of the pertinent parameters on the flow quantities, a parametric study has been conducted. Effects of the various parameters like, magnetic M, non-uniform heat source/sink (A*, B*), velocity ratio parameter(λ), Williamson parameter (W), Lewis number (Le), Unsteadiness parameter (A), Brownian motion Nb, thermophoresis Nt, Prandtl number (Pr) on velocity, temperature and concentration profiles has been clearly analyzed. The numerical results are displayed graphically in tables and got good agreement with the previous results. Tables 1 and 2 displays the values of skin friction coefficient, Nusselt Number for various governing parameters. Table 3 shows the Comparision for $f''(0)$ for different values of velocity ratio parameter.

Fig 1 depicts the effect of the velocity ratio parameter on the flow field velocity. It can be observed that for λ< 1 i.e., when the stretching velocity of the sheet exceeds the free stream velocity, the velocity of the fluid and the boundary layer thickness increases with an increase in λ. Moreover, for λ > 1 i.e., when the free stream exceeds the stretching velocity, the flow velocity increases and the boundary layer thickness decreases with an increase in λ. If λ = 1 i.e., when the stretching and free stream velocities are equal, then there is no fluid flow near the sheet.

Figure 2 indicates the effect of magnetic parameter M on velocity profiles. The velocity of the fluid decreases with an increase in M. This causes retarding effect on the flow field leading to the prominent reduction in velocity due to Lorentz force effect. Therefore, the Lorentz force increases the opposition of the flow of fluid reducing the velocity of the flow.

Fig 3 reveals the effect of Williamson parameter W on velocity profile. It can be observed that velocity decreases with increase in Williamson fluid parameter W; because after increasing Williamson fluid parameter W the fluid offers more resistance to flow which decreases velocity.

Fig 4 shows the effect of unsteadiness parameter A on velocity profile. It can be noticed from this figure that the velocity profile decreases with increase in the unsteadiness parameter A. This is due to the fact that the momentum boundary layer thickness decreases as A increases. Since the fluid flow is merely due to stretching of the sheet so velocity decreases with increase in η till it satisfies the boundary condition at $\eta = \infty$.

Fig 5 is a plot of temperature distribution with η for various values of unsteadiness parameter A. Here the temperature profile increases with increase in the unsteadiness parameter A. Hence the thermal boundary layer thickness increases as A increases.

Fig 6 shows the temperature profile for various values of Pr. It is clear that the dimensionless parameter θ decreases with the increase in Prandtl number. Since the Prandtl number is the ratio of momentum diffusivity to thermal diffusivity; it reduces the thermal boundary layer thickness. In general the Prandtl number is used in heat transfer problems to reduce the relative thickening of the momentum and the thermal boundary layers.

Fig 7 depicts the effect of space-dependent heat source/sink parameter A* on temperature distribution with η in the thermal boundary layer. It is observed that the thermal boundary layer generates the energy, which causes the magnitude of temperature profiles to increase with the increasing values of A* > 0, (heat source) whereas in the case of A*< 0 (absorbtion) thermal boundary layer absorbs energy resulting in the temperature to fall considerably with decreasing the value of A*< 0. The effect of temperature-dependent heat source/sink parameter B* on heat transfer is demonstrated in Fig 8. This graph illustrates that energy is released when B* > 0 which causes the temperature to increase, whereas the energy is absorbed by decreasing the values of B*< 0 resulting in the temperature to drop significantly near the boundary layer.

Figure 9 illustrates the influence of Brownian motion parameter Nb on concentration profile. As the effect of Brownian motion increases, the concentration gradient increases, and as a result, the Brownian force increase boots the nanoparticle concentration at the surface. Hence, the concentration profile φ(η) increases at the surface.

Figure 10 shows the impact of thermophoresis on concentration profile φ(η). Since the impact of Brownian force is to counterbalance the influences of thermophoretic force, as the influences of thermophoretic force increase the concentration gradient at the surface decreases, as a result of this, the concentration profile at a surface decreases, which is opposite to the case for Brownian motion effect.

Figure 11 shows the impact of convective heating called Biot number δ on temperature profile. Physically, Biot number is the ratio of convection at the surface to conduction within the surface of a body. As the effect of Biot number (convection at the surface) increases, temperature at the surface increases; this again results in an increase in the thermal boundary layer thickness.

Figure 12 shows the effect of Biot number δ on concentration profile. As the convection at the surface δ increases the concentration near the surface decreases.

Table 1: Values of Skinfriction coefficient $f''(0)+\frac{W}{2}(f''(0))^2$ for $Pr = 0.71, Nb = 0.2, Nt = 0.5, E = 0.5, M = 1, Le = 5, \delta = 5, A^* = B^* = 0.01.$

A	W=0.0	W=0.1	W=0.2
0.1	-0.83213	-0.81977	-0.80662
0.2	-0.85314	-0.8405	-0.82672
0.4	-0.87439	-0.86090	-0.84648
0.6	-0.89504	-0. 88097	-0.86591
0.8	-0.91538	-0.90072	-0.88500
1.0	-0.93541	-0.92016	-0.90377

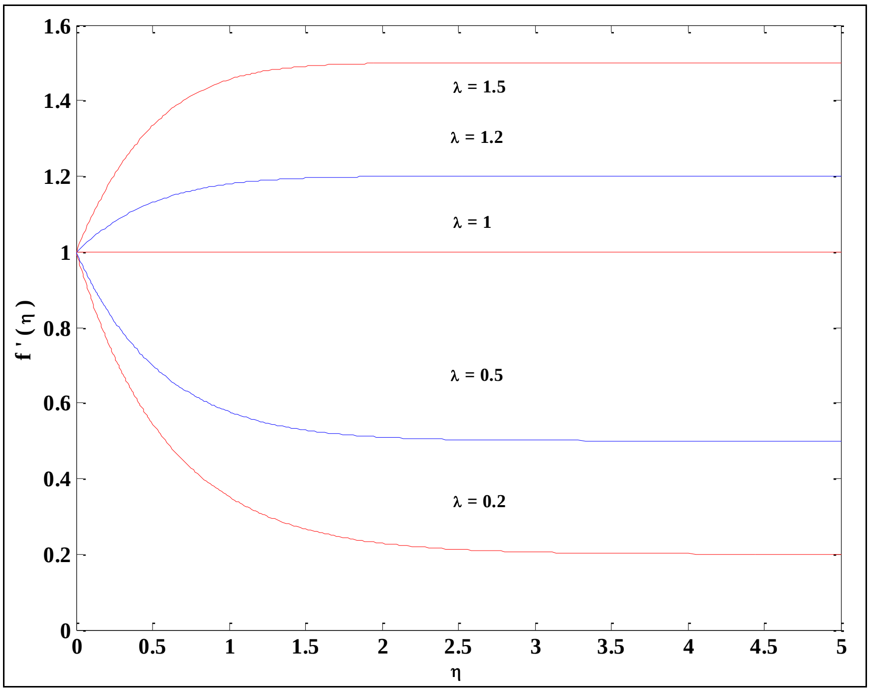

Fig 2: Velocity profile for various values of λ

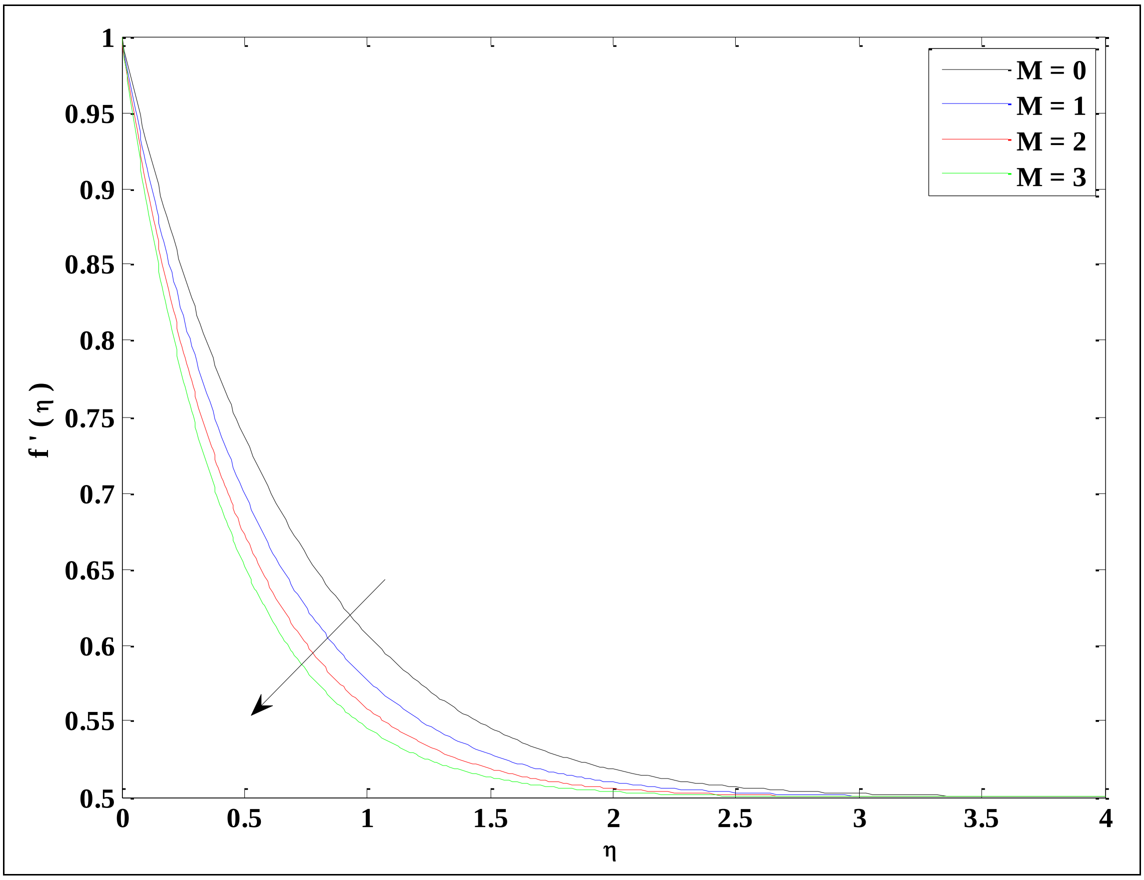

Fig 3: Velocity profile for various values of M

Table 2: Values of Nusselt Number $-\theta'(0)$

$A = 0.5, Nb = Nt = 0.3, A* = 0.02, B* = 0.02, Le = 5, \delta = 5, M = 1, \lambda = 0.5, W = 0.2$

A	Nt	A*	B*	δ	$-\theta'(0)$
0.1 0.3 0.7					0.40210 0.35455 0.21749
	0.1 0.5 1				0.30752 0.28547 0.25918
		0.01 0.05 0.07			0.32575 0.20748 0.14753
			0.01 0.05 0.07		0.30957 0.25319 0.22074
				0.1 1.0 2.0	0.06663. 0.23265 0.26893

Table 3: Comparision table for $f''(0)$ for different values of velocity ratio parameter

A	Wubshet *et al* [33]	Ishak *et al* [34]	Present result
0.1	-0.9694	-0.9694	-0.96939
0.2	−0.9181	−0.9181	-0.91811
0.3	--	--	-0.84942
0.4	---	---	-0.76533
0.5	−0.6673	−0.6673	-0.66726
1.0			0.00000
2.0	2.0175	2.0175	2.01750
3.0	4.7293	4.7293	4.72928

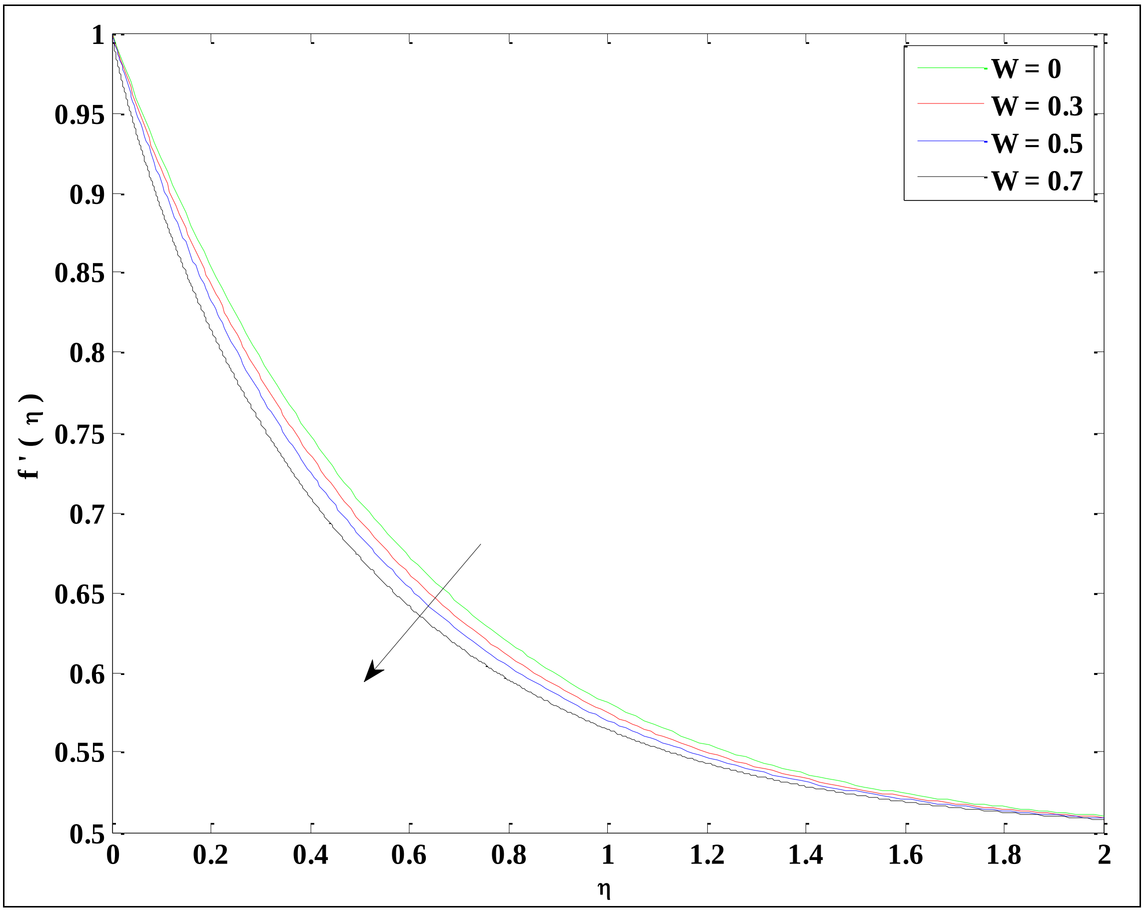

Fig 4: Velocity profile for various values of W

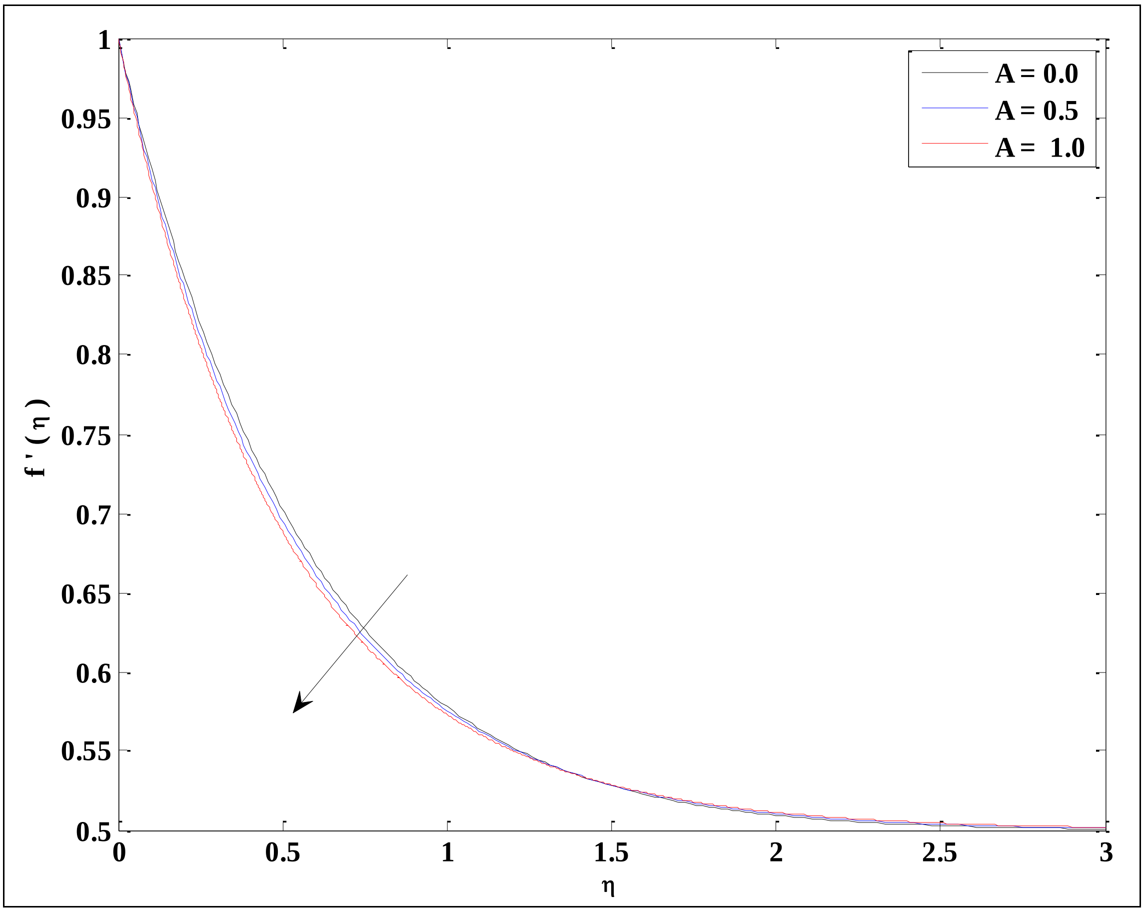

Fig 5: Velocity profile for various values of A

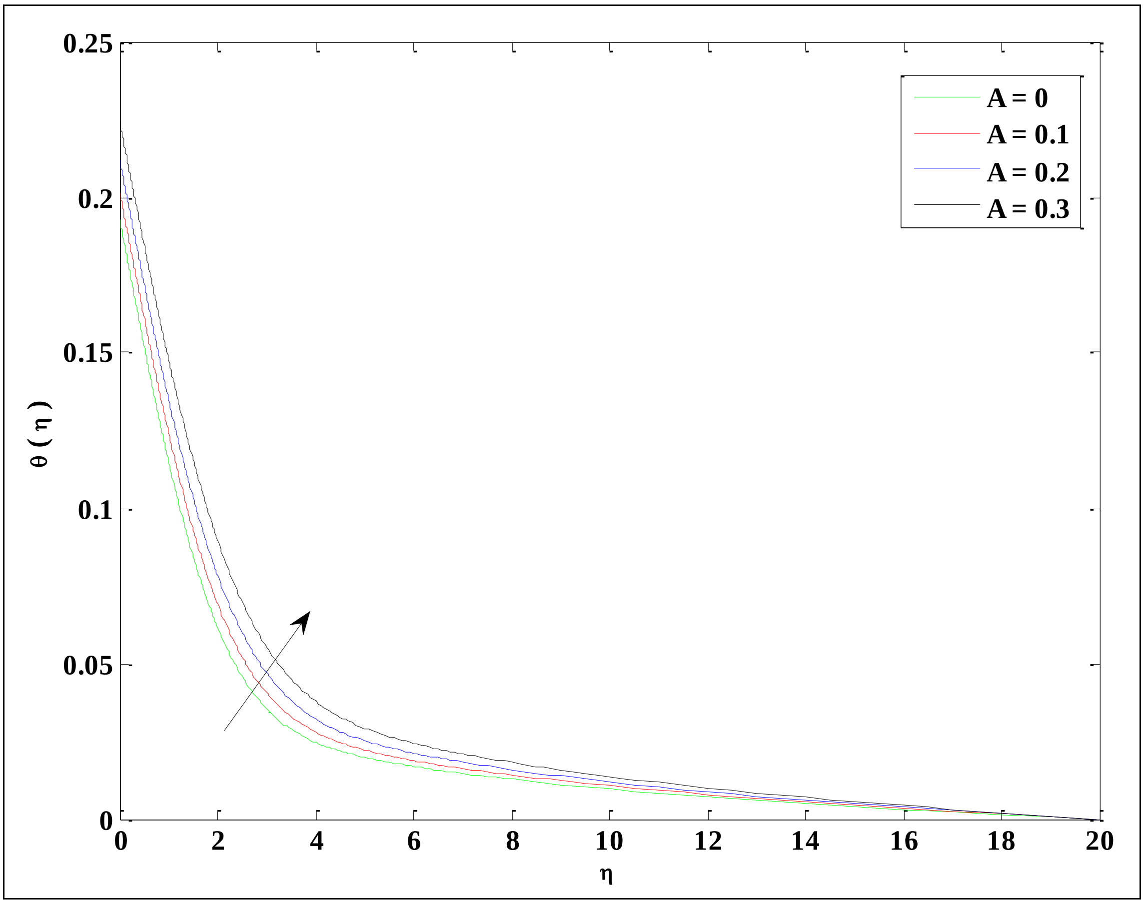

Fig 6: Temperature profile for various values of A

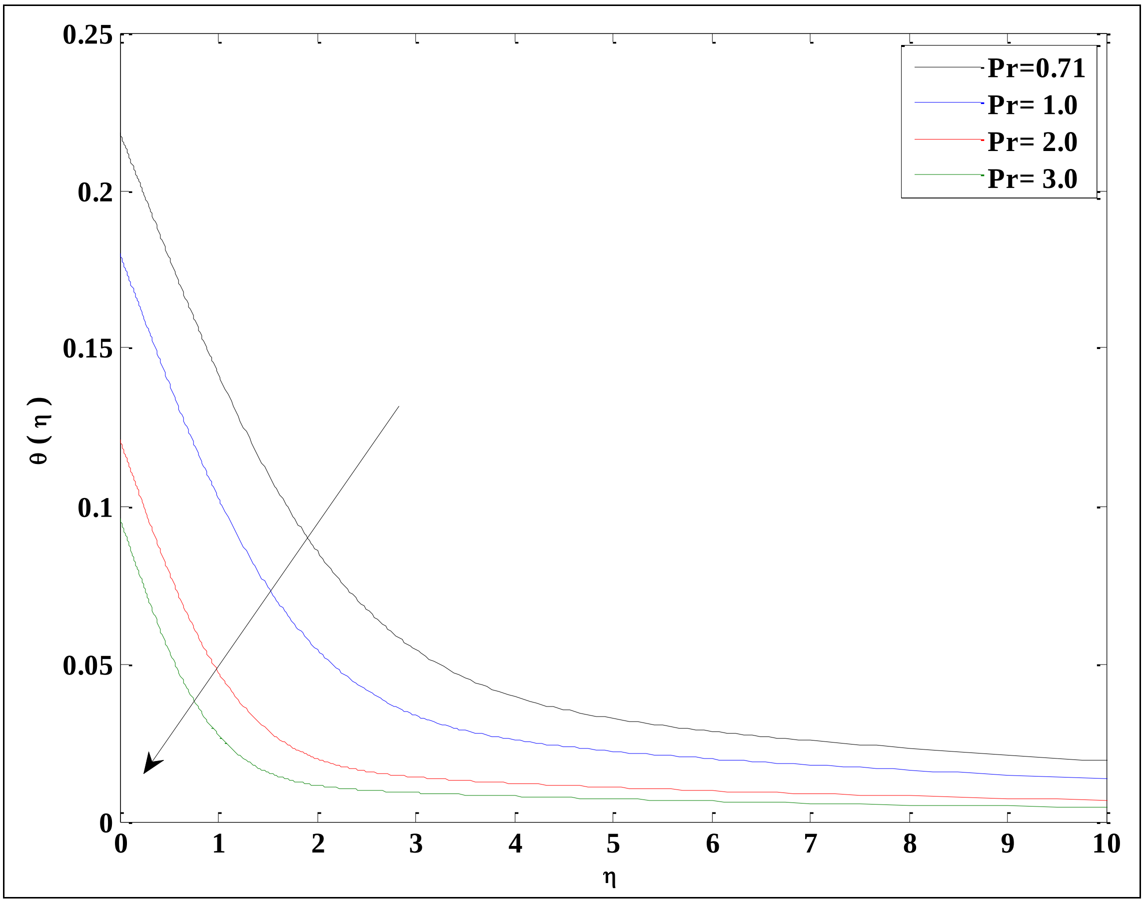

Fig 7: Temperature profile for various values of Pr

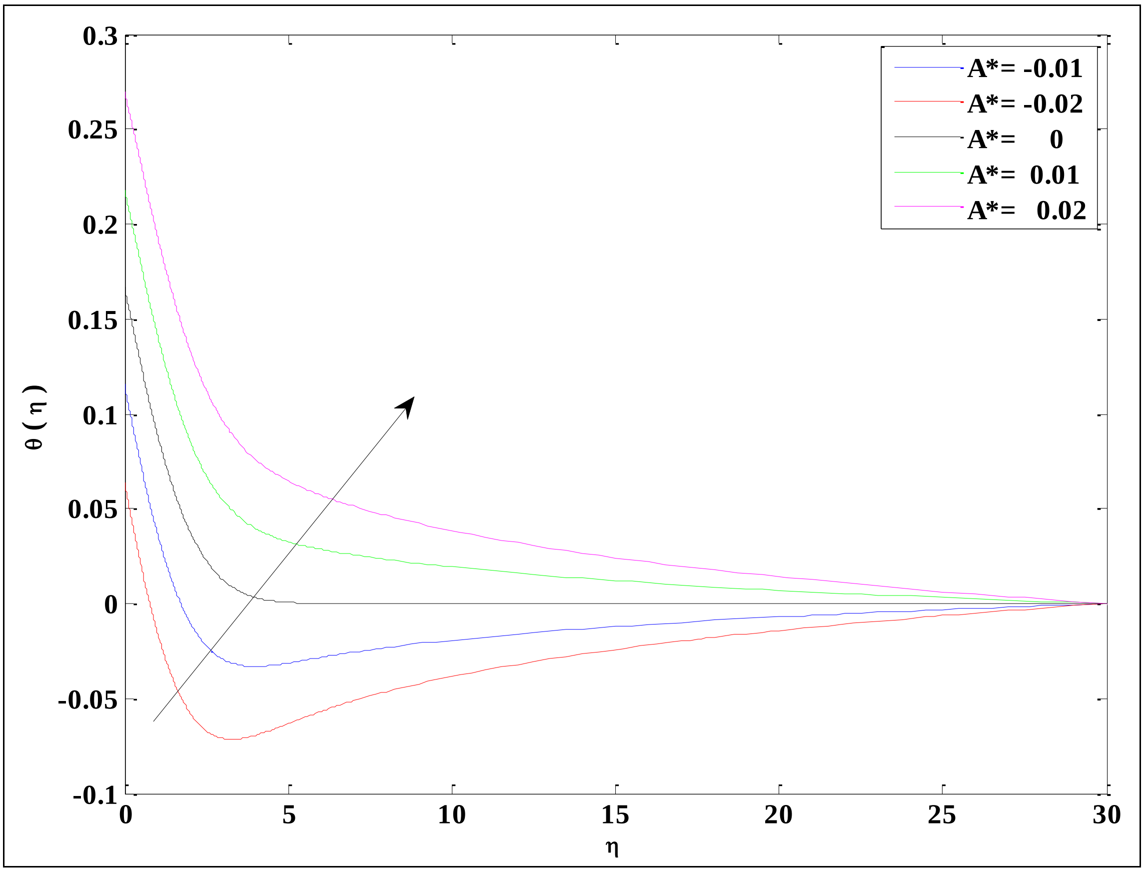

Fig 8: Temperature profile for various values of A*

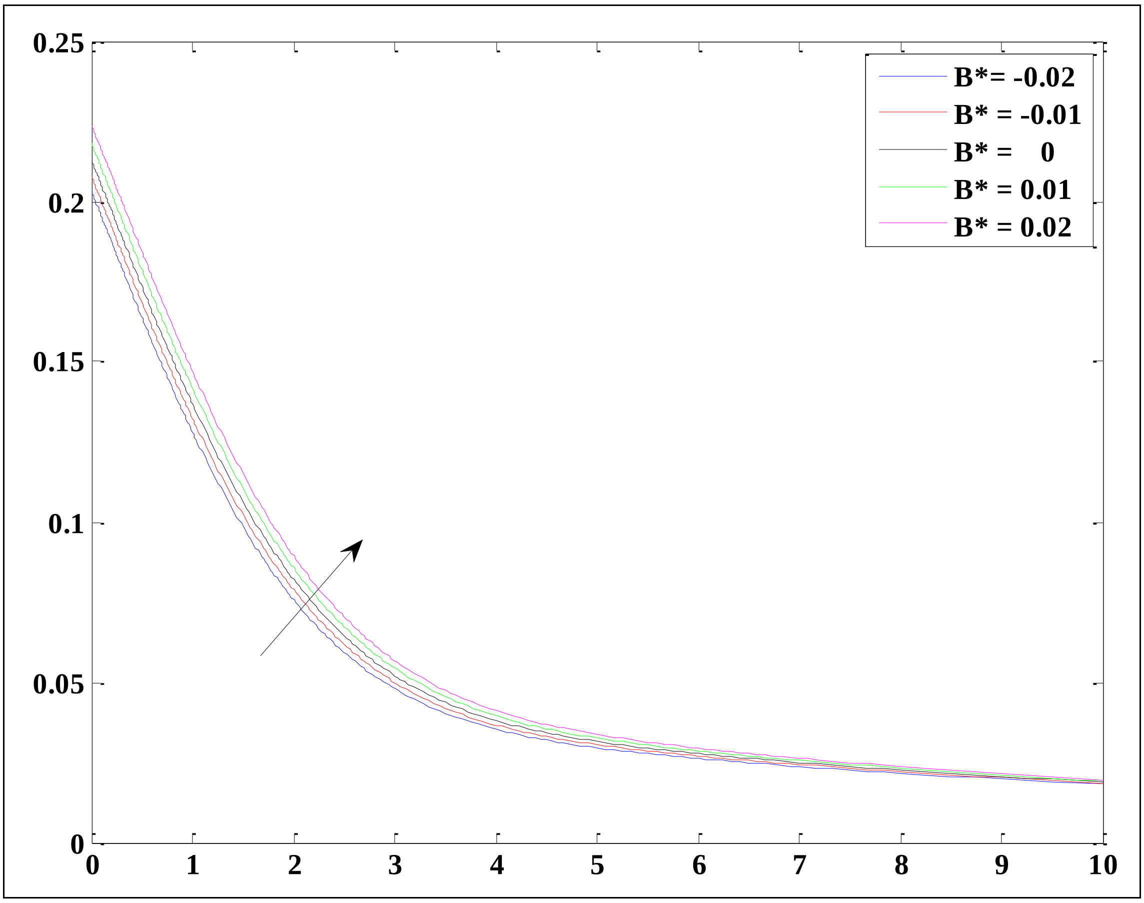

Fig 9: Temperature profile for various values of B*

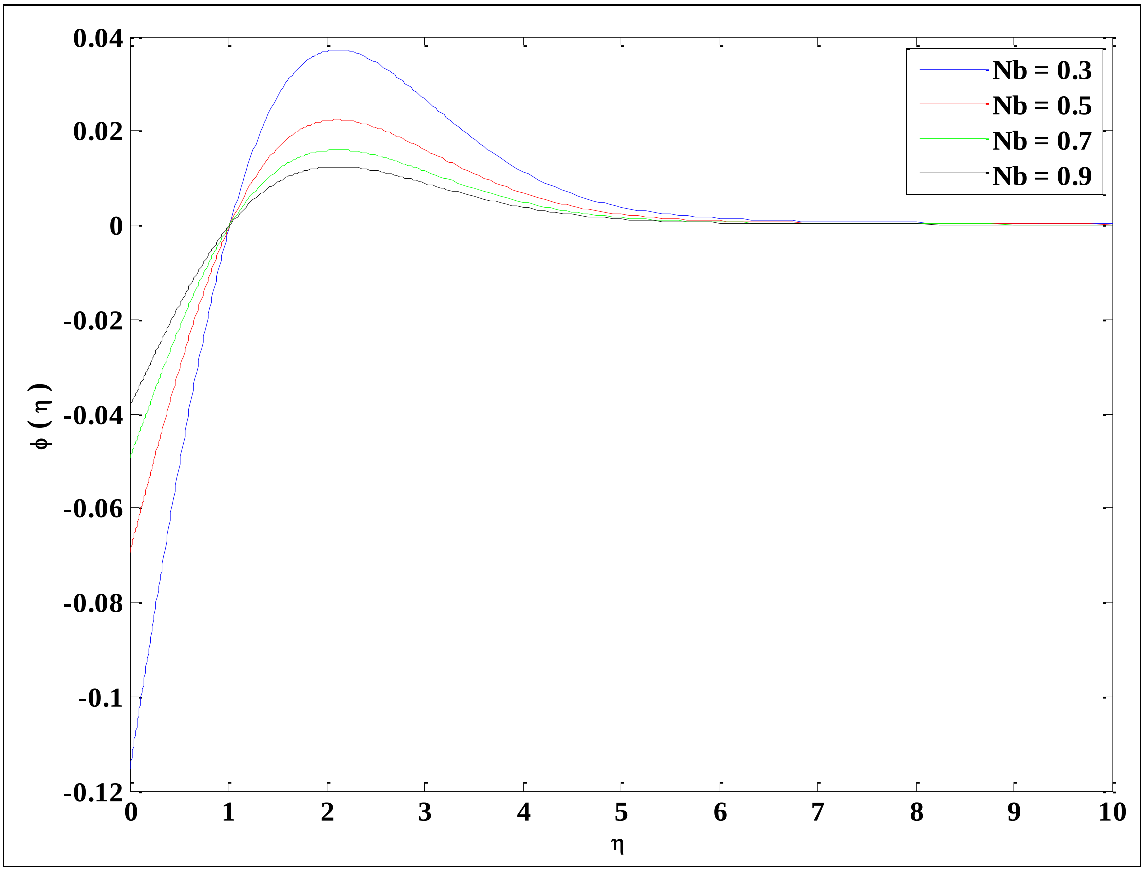

Fig 10: Concentration profile for various values of Brownian motion Nb

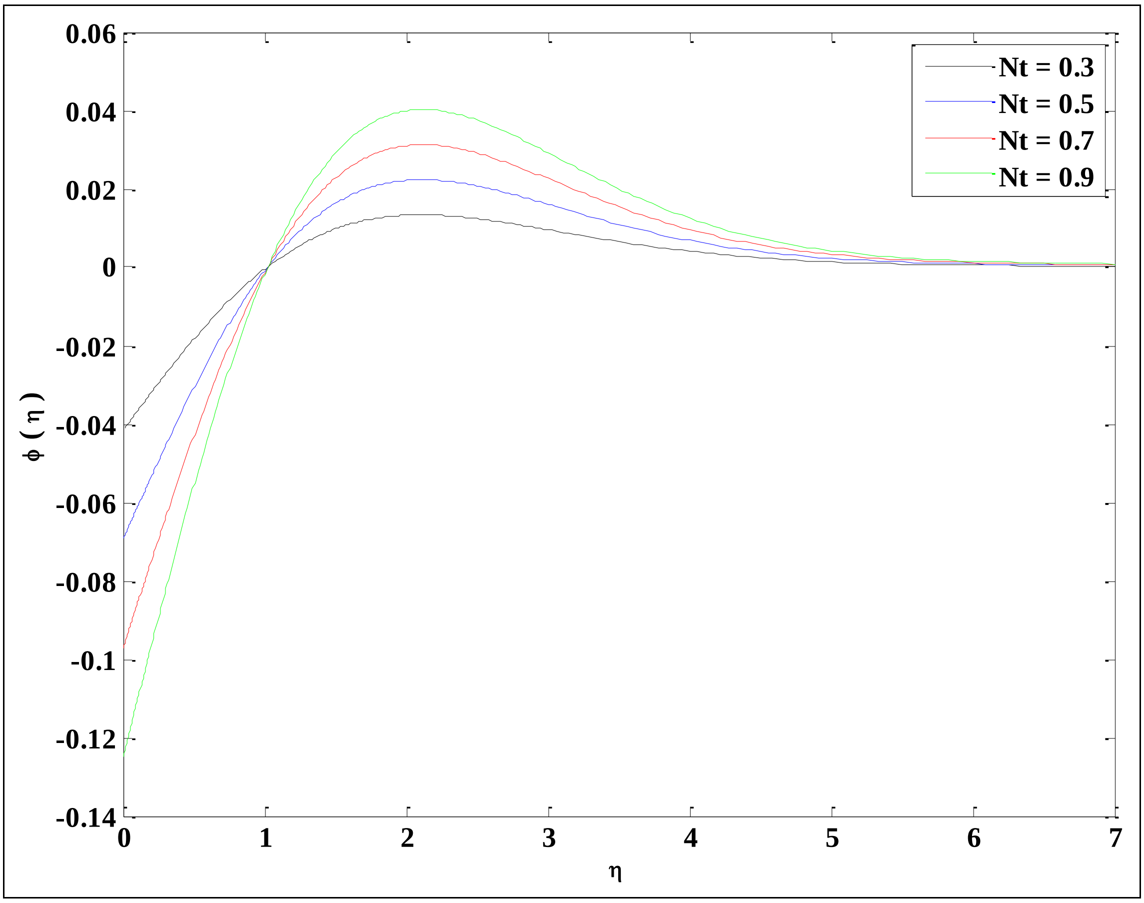

Fig 11: Concentration profile for various values of thermophoresis parameter Nt

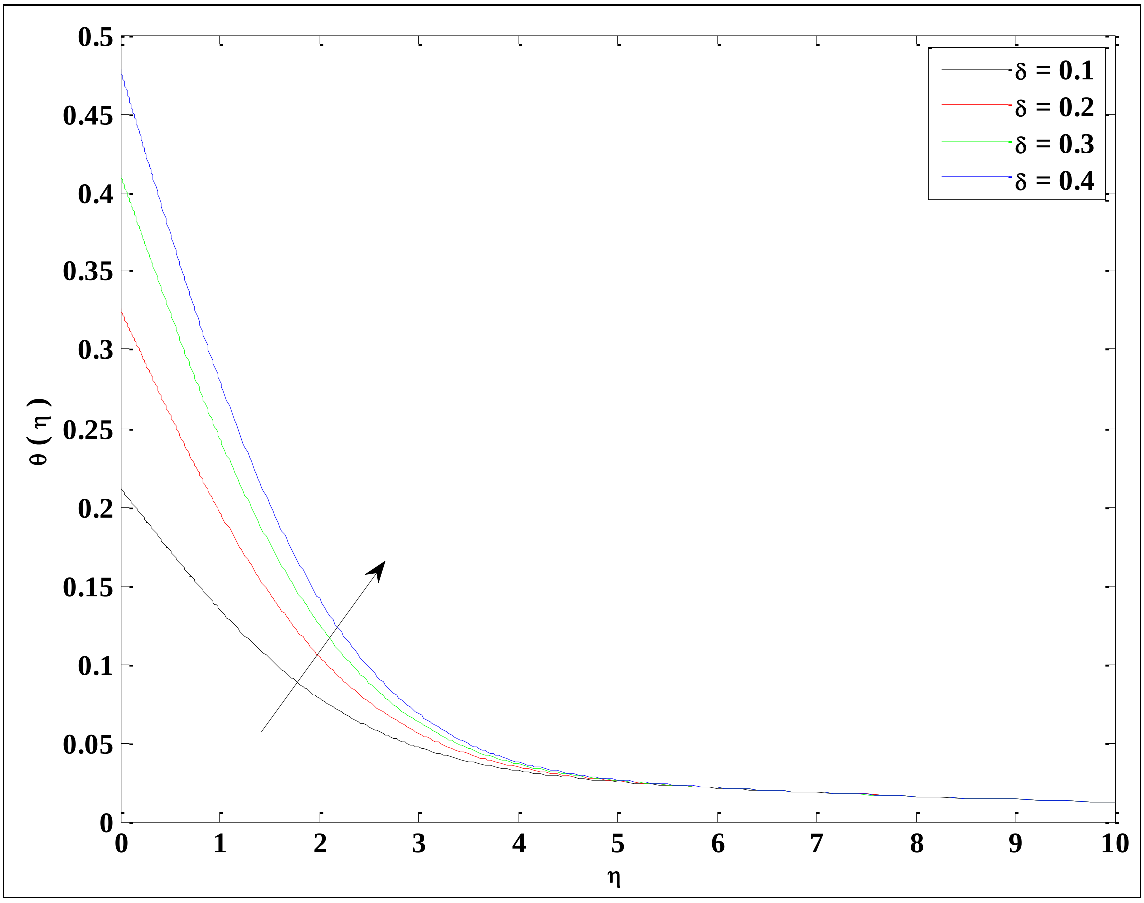

Fig 12: Temperature profile for various values of Biot number δ

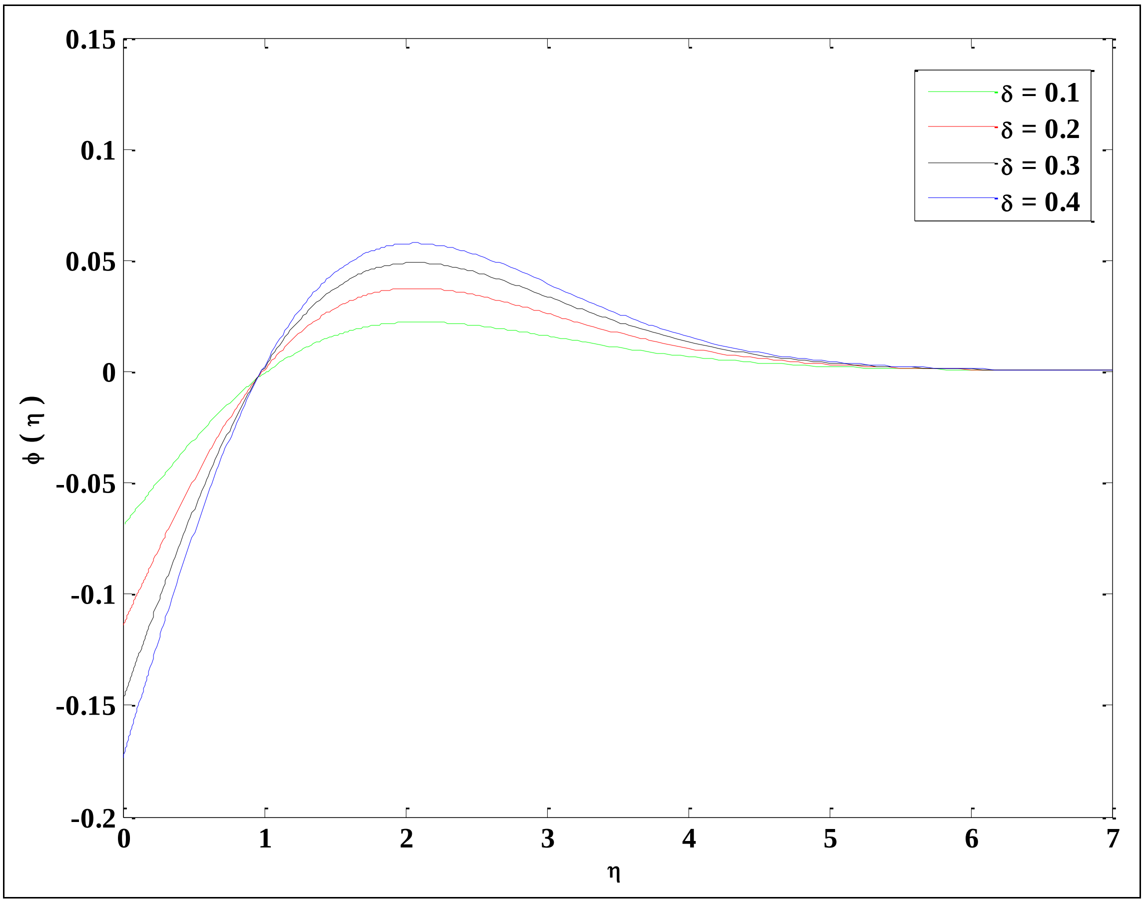

Fig 13: Concentration profile for various values of Biot number δ

Conclusion

In the present study, MHD stagnation point flow of a Williamson nanofluid over a unsteady stretching sheet with convective boundary conditions and non-uniform heat source/sink. are investigated. The numerical solution is obtained by Runge-Kutta-Fehlberg-45 method. The effects of various governing parameters on heat flow characteristics were analyzed. Briefly the above discussion can be summarized as follows.

- The velocity boundary layer thickness reduces for magnetic parameter M.
- An increase in Williamson fluid parameter λ caused the velocity of the fluid to decrease.
- The velocity of the fluid and the boundary layer thickness increases with an increase in λ for $\lambda < 1$ and velocity increases and the boundary layer thickness decreases with an increase in λ for $\lambda > 1$.
- Momemtum boundary layer thickness decreases and thermal boundary layer thickness increases with an increase in unsteadiness parameter A.
- Prandtl number decreases the thermal boundary layer thickness.
- The concentration gradient increases with the Brownian motion whereas the opposite effect is observed for thermophoresis effect.
- The effect of temperature dependent heat source/sink parameters is to generate temperature for increasing positive values and absorb temperature for decreasing negative values.
- The effect of Biot number increases the temperature profile whereas the concentration decreases.

References

1. Hiemenz K. Dinglers Polytech. J,1911:326:321.
2. Howarth L. Proc. Roy. Soc. London A,1938:164:547.
3. Gersten K, Papenfuss HD, Gross JF. Int. J. Heat Mass Transfer,1978:21:275.
4. Crane LJ. Flow past a stretching plate. J Appl Math Phys (ZAMP),1970:21:645-7.
5. Mc Cormack PD, Crane LJ. Physical Fluid Dynamics. New York: Academic Press; 1973
6. Gupta PS, Gupta AS. Heat and mass transfer on a stretching sheet with suction or blowing. Can J Chem Engg,1977:55:744-6.
7. Chakrabarti A, Gupta AS. Hydromagnetic flow and heat transfer over a stretching sheet. Quart Appl Math,1979:37:73-78.
8. Wang CY. The three dimensional flow due to a stretching flat surface. J Phys Fluids,1984:27:1915-17.
9. Andersson HI. MHD flow of a viscoelastic fluid past a stretching surface. Acta Mech,1992:95:227-30.
10. Char MI. Heat and Mass Transfer in a Hydromagnetic Flow of the Viscoelastic Fluid over a Stretching Sheet,1994:186(3):674-689.
11. Chiam TC. "Stagnation-point flow towards a stretching plate". J Phys Soc Jpn,1994:63:2443-4.
12. Mahapatra TR, Gupta AS. "Magneto hydrodynamic stagnation point flow towards a stretching sheet". Acta Mech,2001:152:191-6.
13. Ashraf M, Ashraf MM. "MHD stagnation point flow of a micropolar fluid towards a heated surface", Appl. Math. Mech. -Engl. Ed,2011:32(1):45-54.
14. Pop I, Na TY. "Unsteady flow past a stretching sheet", Mech. Res. Comm., 23, 413-422,.
15. Wang CY, Du G, Miklavcic M, Chang CC. "Impulsive stretching of a surface in a viscous fluid", SIAM J. Appl. Math,1997:57:1-14.
16. Elbashbeshy EMA, Aldawody DA. "Effect of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a porous stretching surface". Int. J. Nonlinear Sciences,2010:9(4):448-454.
17. Swati Mukhopadhyay, Prativa Ranjan De, Krishnendu Bhattacharyya, Layek GC. "Casson fluid flow over an unsteady stretching surface". Ain Shaums Enn. J,2013:4:933-938.
18. Kalidas Das, Pinaki Ranjan Duari, Prabir Kumar Kundu. "Nanofluid flow over an unsteady stretching surface in presence of thermal radiation". Alexandria Eng,2013:53:737-745.
19. Gireesha BJ, Manjunatha S, Bagewadi CS. "Unsteady hydromagnetics boundary layer flow and heat transfer of dusty fluid over a stretching sheet", Afr. Mat,2012:23:229-241.
20. Shijun Lio. "An analytical solution of unsteady boundary-layer flows caused by an impulsively stretching plate", Comm. in Non. Sci. and Num. Sim,2006:11:326-339.
21. Choi S. Enhancing thermal conductivity of fluids with nanoparticle in: D.A. Siginer, H.P. Wang (Eds.), "Developments and Applications of Non-Newtonian Flows", ASME MD,1995:231:66:99-105.
22. Masuda H, Ebata A, Teramae K, Hishinuma N. "Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles", Netsu Bussei,1993:7:227-233.
23. Eastman JA, Choi SUS, Li S, Yu W, Thompson LJ. "Anomalously Increased Effective Thermal Conductivities Containing Copper Nanoparticles," Appl. Phys. Lett.,2001:78:718-720.
24. Kuznetsov AV, Nield DA. "Natural Convective Boundary-Layer Flow of a Nanofluid Past a Vertical Plate," Int. J. Thermal Sci.,2010:49:243-247.
25. Khan WA. Pop I. "Boundary layer flow of nanofluid past a stretching sheet. Int.J.Heat and mass Transfer",2010:53:2477-2483.
26. Hassani M, Mohammad M Tabar, Nemati H, Domairry G, Noori F. "An analytical solution for boundary layer flow of nanofluid past a stretching sheet", Int. J.Thrm. Sci,2011:50:2256-2263.

27. Ibrahim W, Shankar B, Nandeppanavar MM. “MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet”, Int. J. Heat and mass tranf,2013:56:1-9.
28. Wubshet Ibrahim, Bandari Shankar. “MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions”. Computer & Fluids,2013:75:1-10.
29. Nadeem S, Rizwan Ul Haq. “Effect of Thermal Radiation for Megnetohydrodynamic Boundary Layer Flow of a Nanofluid Past a Stretching Sheet with Convective Boundary Conditions”, Journal of Computational and Theoretical Nanoscience,2014:11:32-40.
30. Ajeeb Alam Khan, Hassam Khan. “A Boundary layer flows of non-Newtonian Williamson fluid”, Nonlinear Engineering,2014:3(2):107-115.
31. Nadeem S, Hussain ST, Changhoon Lee. “Flow of a Williamson fluid over a stretching sheet”, Brazilian Journal of Chemical Engineering, 30, 619-625.
32. Nadeem S, Hussain ST. “Heat transfer analysis of Williamson fluid over exponentially stretching surface”, Appl.Math and Mech,2014:35:489-502.
33. Ishak A, Nazar R, Pop I. “Mixed convection boundary layers in the stagnation point flow towards a stretching vertical sheet”. Meccanica,2006:41:509-518.
34. Ibrahim W, Shankar B. Boundary-layer flow and heat transfer of nanofluid over a vertical plate with convective surface boundary condition. J Fluids Eng ASME,2012:134(8):081203.