

A short investigation on fuzzy linear programming problems

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Abstract

Fuzzy logic is a modelling method, a powerful tool for decision-making systems, such as expert and pattern classification systems. A large number of developments have been made so far in Fuzzy sets which played a vital role in fuzzy linear programming problems. In this literature, we survey on methods and techniques for solving fuzzy linear programming. Some of these methods provide fuzzy solution and the others provide crisp solution.

Keywords: Fuzzy sets, Fuzzy linear programming problems

1. Introduction

Fuzzy set theory has been studied extensively over the past 40 years. Most of the early research in fuzzy set theory pertained to representing uncertainty in human cognitive processes (Zadeh (1965) [32]). Fuzzy set theory is now applied to problems in the fields of engineering, business, medical and related health sciences, and the natural sciences. The theory proposes a mathematical technique for handling imprecise concepts and problems that have many possible solutions. Fuzzy sets representing linguistic concepts such as low, medium, high etc., are employed to define states of a variable. Such a variable is known as fuzzy variable. The relevance of fuzzy variables is that they facilitate gradual transitions between states and consequently possess a natural capability to express and deal with the uncertainties in observation and measurement. Traditionally, computation involves manipulation of numbers and symbols. However, now-a-days humans employ mostly words in the natural language for computing and reasoning. A key aspect of computation with words is that it involves the combination of natural languages, and computation is done with fuzzy variables. The notion of a granule plays a vital role in computing with words. According to Zadeh (1965) [32], 'granulation plays a key role in human cognition. For humans, it serves as a way of achieving data comparison'.

Fuzzy sets defined on a set of real numbers R , have great importance. Membership function $\mu : R \rightarrow [0, 1]$ possesses a quantitative meaning and may be viewed as fuzzy numbers provided they satisfy certain conditions. One such condition is the initiative conceptions of approximate numbers or intervals, such as 'numbers that are close to five' or 'numbers that are around the given real numbers'. Such notions are essential for characterizing the states of fuzzy variables. Fuzzy numbers play an important role in many applications such as fuzzy control, decision-making, approximate reasoning and optimization. A fuzzy number is the fuzzy subset of the real line, where the highest membership values are clustered around a given real number.

2. Fuzzy Linear Programming Problems and its Methods

A lot of real world decision problems are described by linear programming models and sometimes it is necessary to formulate them with elements of imprecision or uncertainty.

Therefore, it is useful to consider the knowledge of experts about the parameters as fuzzy data by Zadeh (1965) [32]. Using the concept of decision making in fuzzy environment given by Bellman and Zadeh (1970) [5], Tanaka *et al.*, (1973) [30] proposed a method for solving fuzzy mathematical programming problems. Zimmerman (1978) [33] developed a method for solving fuzzy linear programming (FLP) problems using multiobjective LP technique. Afterwards, many researchers have considered various types of the FLP problems and proposed several approaches for solving FLP problems. In particular, the most convenient methods are based on the concept of comparison of fuzzy numbers with help of ranking functions. Usually in such methods, authors define a crisp model which is equivalent to the FLP problem and then use optimal solution of the model as the optimal solution of the FLP problem. Campos and Verdegay (1989) [8] proposed a method to solve FLP problems involving fuzzy coefficients in both matrix and right hand of the constraints. Inuiguchi *et al.*, (1990) [16] used the concept of continuous piecewise linear membership function for solving FLP problems. Cadenas and Verdegay (1997) [7] solved a LP problem in which all its elements are defined as fuzzy sets. Fang *et al.*, (1999) [13] developed a method for solving LP problems with fuzzy coefficients in constraints. Buckley and Feuring (2000) [6] proposed a method to find the solution for a fully fuzzy linear programming (FFLP) problem by changing the objective function into a multiobjective LP problem. Maleki *et al.*, (2000) [22] solved the LP problems by the comparison of fuzzy numbers in which all decision parameters are fuzzy numbers. Liu (2001) [20] introduced a method for solving FLP problems based on the satisfaction degree of the constraints. Maleki (2002) [23] proposed a method for solving FLP problems with fuzzy constraints by using ranking function. Zhang *et al.*, (2003) [34] introduced a method for solving FLP problems in which coefficients of objective function are fuzzy numbers. Nehi *et al.*, (2004) [25] developed the concept of optimality for FLP problems by transforming into multiobjective LP problems. Ramik (2005) [27] proposed the FLP problems based on fuzzy relations. Ganesan and Veeramani (2006) [14] proposed an approach for solving FLP problems involving symmetric trapezoidal fuzzy numbers without converting it into crisp LP problems. Hashemi *et al.*, (2006) [15] introduced a

two phase approach for solving FFLP problems. Dehghan *et al.*, (2006) ^[9] proposed a FLP approach for finding the exact solution of fully fuzzy linear system of equations which is applicable only if all the elements of the coefficient matrix are non-negative fuzzy numbers.

Jimenez *et al.*, (2007) ^[18] developed a method using fuzzy ranking method for solving FLP problems where all the coefficients are fuzzy numbers. Mahdavi-Amiri and Nasserri (2007) introduced a dual simplex algorithm for solving FLP problems and it's dual by using a general linear ranking function and LP directly. Allahviranloo *et al.*, (2008) ^[2] proposed a method based on ranking function for solving FFLP problems. Nasserri (2008) ^[24] proposed a new method for solving FLP problems by fuzzy ranking method for converting the fuzzy objective function into crisp objective function. Ebrahimnejad and Nasserri (2009) ^[10] solved the FLP problems with fuzzy parameters by using the complementary slackness theorem. Lotfi *et al.*, (2009) ^[21] proposed a new method to find the fuzzy optimal solution of FFLP problems with equality constraints which can be applied only if the elements of the coefficient matrix are symmetric fuzzy numbers and the obtained solutions are approximate but not exact.

Ebrahimnejad *et al.*, (2010) ^[11] proposed a new primal-dual algorithm for solving FLP problems by using duality theorems. Amit Kumar *et al.*, (2010; 2011) ^[3, 4] proposed a method for solving the FFLP problems by using fuzzy ranking function in the fuzzy objective function. Nasserri *et al.*, (2012) considered a kind of linear programming which includes the triangular fuzzy numbers in its parameters and proposed a revised simplex algorithm for an extended linear programming problem which is equivalent to the original FLP problems. Ebrahimnejad and Nasserri (2012) defined a new dual problem for the FLP problems with trapezoidal fuzzy variables and deduced the duality results such as weak duality, strong duality, and complementary slackness theorems. Amit Kumar *et al.*, (2012) proposed a new method for finding the optimal solution to FLP problems and also the advantages of proposed method over existing method were discussed. Barkha Sharma and Rajendra Dubey (2012) discussed FLP problems with the help of simplex algorithm and crisp linear system of equation using the linear ranking function involving trapezoidal fuzzy number. Jayalakshmi and Pandian (2012) ^[17] proposed a bound and decomposition method to find an optimal fuzzy solution for FFLP problems.

Recently, Hashem (2013) proposed a new method for solving FLP problems by converting into classical LP problems using different ranking functions and comparing the results. Hashem (2013) introduced a method for solving FLP problems by converting it into crisp multi objective LP problems. Masoud Sanei (2013) introduced a new method for solving a special kind of fuzzy number linear programming problem with bounded variables directly using linear ranking function. Hashem (2013) proposed a new method for solving FLP problems with non-symmetrical trapezoidal fuzzy numbers by converting them into classical LP problems by using linearity property of ranking function. Nagoor Gani *et al.*, (2013) solved FLP problem by using interior point technique to find the approximate optimal solution. Thangaraj Beaula and Rajalakshmi (2013) proposed a new method to solve FFLP problems using breaking point and duality theory. Nasserri *et al.*, (2013) presented a new novel method for solving FFLP problems which is based on membership function definition

and interactive fuzzy programming solution approach. Uday Sharma (2013) solved FFLP with the help of similarity measure and ranking function, FFLP problem is transformed into crisp nonlinear programming problem.

More recently, Rajarajeswari and Sahaya Sudha (2014) ^[26] proposed a ranking method by converting FLPP into a crisp LP problem with bound variable constraints and then solved by using Robust's ranking technique. Khalifa (2015) ^[19] used a two-level programming approach for solving FLP problems. Se-Ho Oh *et al.*, (2015) ^[29] obtained fuzzy optimal solution is based on the max-min rule by solving crisp LP problem derived from the Zimmermann's symmetric model. Reza Daneshrad and Davood Jafaria (2015) ^[28] introduced a new method by using Ezzati *et al.*, (2014) ^[12], fuzzy arithmetic operation and a fuzzy version of simplex algorithm is proposed for solving FFLP problem whose parameters all are represented by symmetric trapezoidal fuzzy number without converting the given problem into crisp equivalent problem.

3. Conclusion

This paper we have presented the importance of fuzzy linear programming problem with its application. Also we have listed the methods and techniques for solving fuzzy linear programming problem.

4. References

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