International Journal of Advanced Science and Research
ISSN: 2455-4227; Impact Factor: RJIF 5.12
Received: 19-11-2018; Accepted: 24-12-2018
www.allsciencejournal.com
Volume 4; Issue 1; January 2019; Page No. 25-29

Modeling air pollution by use of weibull distribution for sustainable management

Akintunde Oyetunde A
Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti State, Nigeria

Abstract
It was established that air pollution is caused by excessive concentration of one or more contaminants such as dust, smoke, fumes, gases, etc. in the air which adversely affects human health, plants, animals or even damage to properties. These air pollutants are present in the atmosphere as solid, liquid or gaseous substances. The concentration of the air pollutants produces global warming, depletion of ozone layers, acid rain, acidification of soil, surface water and ground water, etc.

Because of the adverse effects in the short- and/or long- term, there is need for proper control and monitor of air pollution both locally and globally. Hence, the research paper then examined the problem of identifying the appropriate distributional form for air pollution concentration data of which Weibull distribution is found to be more appropriate. For the distribution a simple empirical model, which yields approximations to the relative root mean square error of the percentile estimates against sample size and parameter values, was developed and demonstrated. Thus for the distributional model, an estimate of the relative error associated with evaluating high pollutant levels may be readily determined.

Keywords: air pollution, global warming, ozone layers, acid rain, weibull distribution

Introduction
The atmosphere or air is normally composed of approximately 78% nitrogen, 21% oxygen and 1% mixture of carbon dioxide, water vapour and small quantities of inert gases like argon, neon, helium among others. The atmosphere is known to be a significant component of the natural environment because it provides all the gases necessary for the sustenance of all forms of life in the biosphere. The atmosphere also filters the incoming ultraviolent rays, protect the earth's surface and prevent the earth from becoming too hot. As well known, the atmosphere may be divided into four distinct layers having varying characteristics of temperatures and combination of gases as shown in the table below.

Table 1: Major Layers of the Atmosphere

Atmosphere Layer	Altitude Range (km)	Temperature Range (oC)	Important gases
Troposphere	0 - 11	15 to -56	Nitrogen, oxygen, carbon dioxide, water, vapor
Stratosphere	11 - 50	-56 to -2	Ozone
Mesosphere	50 - 85	-2 to -92	Oxygen, nitric oxide, (ionized form)
Thermosphere	85 - 500	-92 to 1200	Oxygen, oxygen atoms, nitric oxide (ionized form)

Source: Encyclopedia Americana, Vol. 2 (International Edition), 1965.

Concept of air pollution
Air pollution is basically the presence of foreign substances in the air. United States Public Health Service defines air pollution as ''the presence in the atmosphere, of one or more contaminants, in such qualities and of such duration as may be or may tend to be injurious to human, plant or animal, life or property, or which unreasonably interfere with the comfortable enjoyment of life or property or conduct of business''. Hence, air pollution can be seen as excessive concentration of one or more contaminants such as dust, smoke, fumes, gases and many others, in the air which adversely affects human health, plants, animals or cause damage to properties. These contaminants (or foreign substances or air pollutions) can be present in the atmosphere as solid, liquid or gaseous substances.

All round the world, five major types of materials are released directly into the atmosphere in their unmodified forms in sufficient quantities to pose a health risk. They are carbon monoxides, hydrocarbons, particulate matters, sulphur dioxide and nitrogen compounds. This group of pollutions is known as *primary air pollutions.* These pollutants may interact with one another in the atmosphere to form new *secondary air pollutants* such as ozone, peroxyaetyl nitrate and other very reactive materials. These air pollutants are emitted into the atmosphere from a large variety of sources among which are shown in Table 2 below

Table 2: Source of Primary Air Pollutant.

Pollutant	Sources
Carbon monoxide	Incomplete burning of fossil fuels Tobacco smoke
Hydrocarbons	Incomplete burning of fossil fuels Tobacco burning Chemicals
Particulate matters	Burning fossil fuels Farming operations Construction operations Industrial wastes Building demolition
Sulplur dioxide	Burning fossil fuels Smelting ore
Nitrogen compounds	Burning fossil fuels

Source: Eldon D. Enger & Bradley F. Smith (2002): Environmental Science: A study of interrelationships.

Agrawal, K.M., Sikdar, P.K., and Deb, S-C (2002) [2], emphasized that the sources of air pollution can be grouped according to the types of sources or number and the spatial distribution of sources. Based on these types of sources, air

pollution can be divided into two groups namely: natural sources and anthropogenic sources. Natural sources include wind-blown dust, pollen grains, sea salt nuclei, volcanic eruptions, forest fire, microbial activities, etc.
Anthropogenic sources include a wide-spectrum at man-made sources of air pollutants which can be further sub-classified into four categories namely:-
1. Industrial sources
2. Domestic sources
3. Motor vehicles and
4. Agricultural sources

Industrial Sources: Combustion of fossil fuels, vis – coal, oil and natural gas in various industries.

Domestic Source: Burning of wood, dung, coal and crop residues in open fires or in stoves for cooking and heating emit clouds of pollutants like smoke, particulate matter carbon monoxide (CO), sulphur IV oxide (SO_2), and nitrogen dioxide (NO_2) into the atmosphere.

Motor Vehicle exhaust: Motor vehicles are a major source of various air pollutants especially in the urban atmosphere, where approximately between 60% and 70% of the air pollution is caused by automobiles and locomotives. The problem is of much concern in Nigeria as the vehicular population, locomotives and generators are increasing at an alarming rate every year.

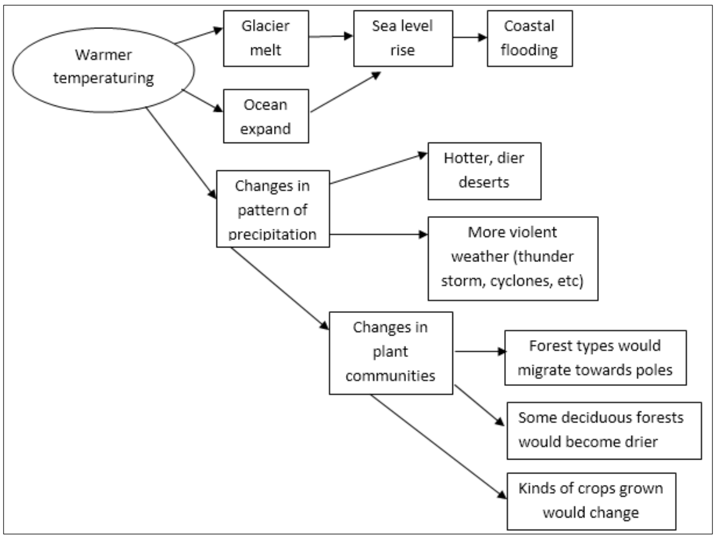
Agricultural Sources: Ploughing up of agricultural fields lead to emission of dust particles and odour in the atmosphere. Use of agricultural chemicals like pesticides, for improvement in crop yield result into emission of nitrates and phosphates, chlorinated hydrocarbons, arsenic and lead particulate matters into the atmosphere. Field refuse and burning for cleaning land for various farming practice also result in emission of pollutants like smoke, fly ash and soot into atmosphere.

Global effects of air pollution

Everyday millions of tones of a variety of toxic air pollutants are released into the atmosphere by natural and man-made activities. These air pollutants are transported to many places which are several thousands of kilometers away from the source through atmospheric circulation systems and thereby cause irreparable damages to various environmental components on both continental and global scales. The major global effects of air pollutions are global warming, depletion of ozone layers, acid rain, acidification of soil, surface water and ground water, etc.

Global Warning

As more air pollutants are emitted, green house gases like carbon dioxide, chlorofluorocarbons, methane will be more transparent to light and absorb infrared radiation. These gases in turn allow sunlight to penetrate the atmosphere and be absorbed by the earth surface. This sunlight is radiated back as infrared radiation (heat), which is absorbed by these gases. As a result of this, the earth's surface and the lower atmosphere become warmer. Singer (1970) [10] estimated that the overall increase in the average temperature of the earth over the past one hundred years has been about 0.3 – 0.6ºC per decade. However, it was also estimated that doubling of the carbon dioxide concentration by 2050 will bring about a 3ºC rise in the surface temperature which in turn may change the climate of the major regions of the world.
The effects of global warming are rising sea–level, worsening health effects, disruption of water cycle, changing forest and natural areas, challenges to agriculture and the food supply, among others and these are as illustrated in the figure below.



***Source*:** Elevations from USGS digital data. (Prepared by the US Geological Survey, 2000)

Fig 1: Effects of global Warming

Depletion of Ozone Layer

Despite ozone is present at almost all altitude of the atmosphere, the bulk of its concentration is present in stratosphere. Ozone layer is however considered as a protective shield or earth's umbrella, because it prevents the harmful ultraviolent solar radiation from reaching the earth's surface. Thus the presence of ozone layer in the stratosphere is of vital significance for all biota including plants, animals and man in the biosphere.
The ozone gas (O_3) is noted to be unstable because it is continuously being created in the stratosphere by the absorption of short-wave length ultraviolent radiation, while

at the same time it is continuously being removed by various chemical reactions that convert it back to molecular oxygen.
The balance between the creation and removal of ozone is being affected by increasing the stratospheric concentrations of chlorine, nitrogen, bromine, hydroxides, etc which act as catalysts, thus speeding up the removal process. One way to describe the catalytic destruction of ozone is with the following set of reactions.

$$X + O_3 \longrightarrow XO + O_2$$
$$\text{And} \quad XO + O \longrightarrow X + O_2$$
$$\text{For a net} \quad O + O_3 \longrightarrow O_2 + O_2$$

Where X may be CI, Br OH or NO

The net result of these pairs of reactions is the destruction of one ozone molecule. The original catalyst that started the reactions, however, may go on to destroy thousands of ozone molecules before it eventually leaves the stratosphere.
The most prominent type of ozone – destructive gases are the chlorofluorocarbons (ClFC). ClFC's are very stable compounds that are relatively unaffected by the usual pollutant removal process in the troposphere. When they drift to the stratosphere the ClFC molecules can be broken down by ultraviolent radiation, thus freeing the chlorine which destroys ozone.
For example CCI_2F_2 is

$$CCI_2F_2 + hv \longrightarrow CI + CCLF_2$$

{hv is the ultraviolent radiation which has wavelength less than 242 nanometre (nm)}

The creation and destruction of ozone is a regular natural process which ought to never disturb the equilibrium level of ozone in the stratosphere. Uberoi (1999) [11] however observed that when the destruction of ozone exceeds the level of ozone creation, serious consequences are bound to crop in due to disequilibrium in the level of ozone. The consequences which will adversely affect life in the biosphere in the following ways:-

1. The increase surface temperature, because of more ultraviolent solar radiation reaching the earth's surface due to ozone depletion would cause skin cancer, retard physiological growth and cause further suppression of mental development of human beings.
2. Increased exposure to ultraviolent radiation would decrease immunity levels and human beings would be more prone to infectious diseases.
3. Photosynthesis, water use efficiency and yield of plants would be marked decreased, and many others.

Acid Rain

Acid rain is caused when coal, fire-wood and petroleum products are burnt and a large quantity of sulphur and nitrogen are released on combustion and rise into the air as SO_2 and NO_2 gases. When these gases are released from industries and power plants, as Agrawal, K.M. *et al* (2002) [2] observed:

1. Up to 50% of the pollutants lingers in the vicinity of the furnace as gas or as particles in air, which eventually drift to the earth within a radius of 30km of the emission source.
2. Up to 30% of the pollutants mix with water in the cloud and fall as localized rain, snow, sleet or mist.
3. The remaining 20% find a convenient wind current and travel a long distance before falling back to the earth as dust, rain, snow, mist.

Naturally, all rainfall is slightly acidic because of a reaction of water vapour with the CO_2 present in the atmosphere. Thus, a pure rain has a pH of 5 - 5.6. So, strictly speaking, acid rain refer to precipitation with pH less than 5. De, A.K. (2000) noted that the acidity arises due to transformation of SO_2 to H_2HSO_4 and of NO_2 to HNO_3.

$$SO_2 \xrightarrow{O} SO_3$$
$$SO_2 + H_2O \longrightarrow H_2S0_{3(aq)}$$
$$SO_3 + H_2O \longrightarrow H_2S0_{4(aq)}$$
$$3NO_2 + H_2O \longrightarrow NO_{(g)} + 2H_2NO_{3(aq)}$$

The environmental consequences of acid rain include acidification of soil, surface water and ground water; damage of the wax-like protective layer of leaves thus making plants more vulnerable to attack by insects, fungi as well as drought, as water loss through leaves increases; removal of nutrients through leaching and retarding activity of decomposers and nitrogen fixing organisms; mobilization of toxic elements like mercury and aluminum from soil which otherwise remain in unavailable form; disfigurations or damage of priceless objects of art and architecture; and many others.

General modelling for air pollution

Air pollution levels depend on the total emissions, transport and transformation phenomena in the atmosphere and deposition processes. According to Moussiopolous, N., Schlunzen H. and Louka P.(2003) [8], analyzing the potential for the practical use of pollution models implies investigating what kind of statements can be made by the aid of models (qualitative approach) and what is the accuracy of these statements (quantitative approach). Apparently, the former approach is easier because it does not require more than understanding the characteristics and the range of application of a model. In addition to that, a quantification of the accuracy of model results presupposes insight into:

1. Input data accuracy and how the data affect the accuracy of model results,
2. Uncertainties in model assumption and parameterizations
3. Methodologies for judging to what extent model results represent reality.

Schlunzen H. (2001, 2002a) [8, 12] listed an inventory of some models which had been developed and applied to air pollution on both rural and urban scale phenomena, among which are:

- ADMS : Atmospheric Dispersion Modeling System
- CAR-FMI : Contaminants in the Air from a Road-Finishing Meteorological Institute
- CALGRID : California Grid Model
- GRAMM : Graz Mesoscale Model
- MARS : Model for the Atmospheric Dispersion of Reactive Species
- MECTM : Mesoscale chemistry and Transport Model
- MEMO : Mesoscale Model
- METRAS : Mesoscale Chemistry, Transport and Stream mode
- MIMO : Microscale Model
- METRAS : Microscale Chemistry, Transport and Fluid (Stream) model.
- MUSE : Multi scale for the Atmospheric Dispersion of Reactive Species
- OFIS : Ozone Fine Structure Model
- VADIS : Pollutant dispersion in the Atmosphere Under Variable Wind Conditions.

Mathematical Modelling of Dispersion

As Salmond J. A., Clarke A. G. and Tomlin A. S. (2006) suggested air quality models are effective tools calculating the concentration of pollutants at a given point in time and space, and can also be used to investigate more about the behavior of different components of the system. These models are based on mathematical representations of the atmospheric processes that determine the rate at which clean air is mixed with polluted air and the pollutants are transported away from the source area. There are a number of techniques for modeling the dispersion and reaction of atmospheric pollutants. The models vary in sophistication, but all include some simplification of atmospheric dispersion processes. More complex models attempt to represent all the physical processes that are relevant in the atmosphere. These are called prognostic models. Simpler tools rely on statistical parameterizations of an individual or a group of processes. These are empirical or statistical models.

Space does not allow us a fully study here but the main methods will be discussed. The choose of a particular type of model depends on many factors such as the temporal and spatial scale of the problem like urban roadside, plume dispersion, regional scale smog modeling, global circulation modeling etc. and the resolution of the data available to run the model and validate the output. The need for models, which include chemical reactions and depositions, should also be considered, as well as external factors such as the skill of the operator, the time available and the financial and computer resources available. Available methods include Gaussian formulations, Eulerian grid modeling, Lagrangian trajectory modeling and turbulence models which try to account for apparently random fluctuations.

The simplest and most common approach is the Gaussian model (otherwise known as a normal or Gaussian distribution). This model assumes that when pollutant concentrations are averaged over a period of time (usually an hour) the peak concentration will occur near the centre of the plume (i.e. the cloud of pollutants that rises and curves upwards in the air) with concentrations dropping in the horizontal and vertical plane according to a bell-shaped curve, The curve is symmetrical about the mean and tends towards zero at the tails. The Gaussian plume dispersion model is based on the assumption that an empirical relationship between atmospheric parameters such as wind speed and direction or stability (which determines dispersion) and the distortion of the Gaussian distribution can be formulated to describe the concentration of pollutants downwind of a source. For a ground level source in very stable conditions, high ground-level concentrations will occur near the source of the pollution whilst in unstable conditions dispersion is much more effective and more dilute concentrations will be measured over a wider area.

Fast screening type calculations of the dispersion of a pollutant from a point source as a large chimney are usually based on the Gaussian plume model. This is because it is a simple, computationally efficient model. It treats dispersion as a statistical process, rather than attempting to represent the individual turbulent motions of the atmosphere. These models typically only require simple input data sources such as wind speed and direction, atmospheric stability class and temperature for example. Gaussian plume models can also be adapted to treat line sources (such as roads) and area sources (such as wind-blown dust from a stockpile or odours from a sewage works). Urban areas can be modeled as a sum of area sources representing domestic and commercial emissions plus larger point sources such as factories or power stations. Gaussian plume models are particularly useful for calculating long term average distributions around a source using statistically averaged meteorological data.

The disadvantage of Gaussian plume models is that they are steady state models. They assume that conditions remain similar over the averaging period. This means that the Gaussian model cannot represent the random fluctuation present in a real plume (for this a turbulence model would be required). In their standard form Gaussian models can only represent first order chemical reaction or deposition processes. This is a significant limitation for secondary pollutants such as ozone whose concentrations depend on a series of non-linear chemical reactions involving NO_x and hydrocarbon species, and levels of sunlight. Gaussian models are therefore best suited to determining averaged concentrations of non-reactive pollutants. They are commonly used as an air quality management tool for primary pollutants from roadways and industrial chimneys.

Weibull distribution and air pollution

Weibull distribution is one of the continuous probability functions. The three-parameter Weibul distribution has the probability density function (p.d.f) given by:

$$f(x/\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma}) = \frac{\boldsymbol{\alpha}}{\boldsymbol{\beta}}\left(\frac{\boldsymbol{x}-\boldsymbol{\alpha}}{\boldsymbol{\beta}}\right)\boldsymbol{exp}\left[-\left(\frac{\boldsymbol{x}-\boldsymbol{\alpha}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha}}\right]$$

With the constraints $-\infty < \boldsymbol{\gamma} < \infty$

$x < \boldsymbol{\gamma}$

$\alpha, \boldsymbol{\beta} > 0$

As Adelakun, A. A. (2006) suggested the three parameters of the Weibull distribution in particular, that are $\boldsymbol{\alpha}$. $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, are used to determine in environmental pollution studies: the location, the shape and the scale respectively of the distribution.

Weibull distribution can also take the form of two-distribution with the probability density function (p.d.f):

$$f(x/\alpha,\beta) = \alpha\beta x^{\beta-1}\exp(-\alpha x^{\beta})$$

With the constraints $x > 0$

$\alpha, \boldsymbol{\beta} > 0$

Weibull introduced the distribution which bears his name principally on empirical grounds (to represents certain life test data). The distribution can take a wide variety of shapes and can therefore be used to model both right and left skewed data set. It worth to be noted that Weibull distribution can be used for a variety of real life analysis and application, among which are:

- In survival analysis
- In reliability engineering and failure analysis
- In industrial engineering to represent manufacturing and delivery times
- In extreme value theory
- In weather forecasting
  - To describe wind speed distributions, as the natural distribution often matches the Weibull shape
  - Fitted cumulative Weibull distribution to maximum one-day rainfalls
- In communications systems engineering

- In radar systems to model the dispersion of the received signals level produced by some types of clutters
- To model fading channels in wireless communications, as the Weibull fading model seems to exhibit good fit to experimental fading channel measurements

- In General insurance to model the size of Reinsurance claims, and the cumulative development of Asbestosis losses
- In forecasting technological change (also known as the Sharif-Islam model)
- In hydrology the Weibull distribution is applied to extreme events such as annual maximum one-day rainfalls and river discharges. The blue picture illustrates an example of fitting the Weibull distribution to ranked annually maximum one-day rainfalls showing also the 90% confidence belt based on the binomial distribution. The rainfall data are represented by plotting positions as part of the cumulative frequency analysis.
- In describing the size of particles generated by grinding, milling and crushing operations, the 2-Parameter Weibull distribution is used, and in these applications it is sometimes known as the Rosin-Rammler distribution. In this context it predicts fewer fine particles than the Log-normal distribution and it is generally most accurate for narrow particle size distributions. The interpretation of the cumulative distribution function is that $F(x; k; \lambda)$ is the mass fraction of particles with diameter smaller than x, where λ is the mean particle size and k is a measure of the spread of particle sizes.

Conclusion and recommendation

Due to the adverse effects in the short- and/or long- term, there is need for proper control and monitor of air pollution both locally and globally. As examined the problem of identifying the appropriate distributional form for air pollution concentration data of which Weibull distribution is found to be more appropriate since it makes use of the location, the shape and the scale of environmental air pollution and the pollutants.

It is therefore recommended from the studies of Weibull distribution that for proper management of air pollution:

- There should be drastic reduction in terms of scale in the consumption of fossil fuels and other sources of air pollution.
- More advanced and efficient technologies should be developed, so that maximum energy may be derived from the existing fossil fuels and the emission of air pollutants at all locations should be minimized.
- Serious efforts should be made to promote reduction in the production and consumption of ozone depleting chemicals.
- Serious efforts should be made to produce and propagate the use of alternative chemicals which do not deplete ozone laye in the stratosphere.

References

1. Adelakun AA. Advanced Distribution Theory. STA 751 Lecture Notes (Unpublished). Department of Statistics, University of Ibadan, Nigeria, 2006.
2. Agrawal KM, Sikdar PK, Deb SC. A Textbook of Environment. Rajiv Beri for Macmillian India Limited, 2002.
3. De AK. Environmental Chemistry. New Delhi: New Age International (P) Ltd, 2000.
4. Eldon D, Enger Bradley F. Environmental Science: A study of interrelationships, 2002.
5. Encyclopedia Americana, Vol. 2 (International Edition), 1965.
6. Moussiopolous N, Scwunzen H, Louka P. Modeling Urban Air Pollution. Air Quality in Cities, 2003.
7. Salmond JA, Clarke AG, Tomlin AS. The Atmosphere in an Introduction to Pollution Science, 2006, 8-71.
8. Schlunzen HK. Model Inventory for EUROTRAC-2 Subproject SATURN.IN: Midgel PM, Reuther M, Williams M (eds) Proc. EUROTRAC-2 Symposium 2000, Springer Verlag, Berlin, 2001.
9. Schlunzen HK. SATURN Model Overview, 2002, http://www.mi.uni_hamburg.de/technische_meteorologies/Meso/Saturn/overview.html
10. Singer SF. Global Effects of Environment Pollution. New York: Springer-Veriag, 1970.
11. Uberoi NK. Environmental Management. New Delhi Publisher Excel Books, 1999.
12. US Geological Survey. Elevations from USGS digital data, 2000.